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Abstract

This paper argues that individuals concerns for relative position contribute to the emergence of development traps. It demonstrates that changes in the mean and the distribution of income qualitatively modify the individuals reference group, by affecting the magnitude of the reference standard, and influence the dynamical transition of the agents' incomes. An increase in mean income and a reduction of inequality cause an increase in the reference standard, inducing, in the long-run, the transition from a Solovian-type stage to a development traps regime as agents need to sacrifice relatively more resources in order to keep up with the reference group.

Keywords: Social inclusion, keeping up with the Joneses, development traps.

JEL: D31, D90, O40

1 Introduction

Holding a relatively advantageous position in the society is not only a source of direct utility benefits, reflecting innate concerns for relative standing, but it also brings about productive advantages that can reinforce individuals concerns for their relative position (Hirsch, 1976).

Exploring this idea, this paper advances the hypothesis that concerns for relative position lead to the emergence of development traps. I propose a modification of the standard keeping up with the Joneses (KUJ) approach, assuming that KUJ attitudes are not hard-wired in the individual preferences, but that they are active due to the productive gains accruing from the social participation in one own reference group. Keeping up with the Joneses is the idea that agents care not only of their absolute but also of their relative level of consumption with respect to that of the people in their reference group. This way of modeling concerns for relative standings implies that the preferences depend negatively on the level of consumption of the individuals in the reference group so that agents engage in a rat race in order to keep up with the benchmark level of consumption. In contrast, I assume that agents strive to consume the benchmark level of a social participation good not only because they have innate concerns for their relative position in the society, but because participating in social groups, or - as I define it - *staying with the Joneses*, brings about a productive informational advantage, which produces a boost in their individual utility[1]. The higher the productivity of staying with the Joneses, the higher the production benefits enjoyed from the interaction with the reference group, and the stronger the incentives in keeping up with it.

Formally, parents' preferences are defined over their consumption of a social participation good and human capital of their children, which is accumulated through two channels; directly, by means of the parents educational expenditures, and indirectly through an informational advantage that parents obtain if they consume the social participation good at the benchmark level of the reference group. Social participation over the reference standard generates an extra utility premium, which results in a discrete jump in the indirect utility function[2]. This implies that whenever the productivity of staying with the Joneses is strong enough, a set of low-middle income agents strictly prefer to completely sacrifice investment in education in order to get the benchmark, instead of sharing the same level of income across both the two goods but not being able to enjoy the extra benefits induced by participation.

In line with the empirical evidence reported in Section 2, this formalization has one main implication.

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[1] "Your ability to enjoy an uncrowded beach may depend on your knowing about that beach when others do not, so that the absolute advantage you will enjoy-being on an uncrowded beach-will depend on your relative position-knowing something that others do not." (Sen, 1983)

[2] As a working example, consider the possibility of going out to have a drink with someone. Increasing the frequency (Duesenberry, 1949) of the meetings induces an improvement in the quality of the relationship, such that at a point in time there is a jump in the satisfaction derived from the relationship; increasing the frequency of the meetings also increases the probability to receive relevantly productive information, due to a network effect. Higher is the productivity gain, much stronger the preference effect is. This example would highlight, moreover, that neither indivisibilities in the characteristics of the good nor restrictions to the access of its consumption (membership, club goods) are needed to generate the discontinuity in the preferences.

The discontinuity generated by the utility premium introduces non-homotheticity only in the preferences of higher income agents, suggesting that the closer the individuals' incomes are to the endogenous threshold, the stronger are the incentives of the agents in changing behavior towards the purchase of this participation good. More specifically, at low levels of income preferences are homotethic so that agents share their budget constraint across the two goods in fixed proportions. As individuals' incomes approach the threshold level of income - the relative poverty line, which discriminates between being part or not of the reference group, concerns for relative position kick in and they are shaped by two parameters; the strength of the concerns and the productivity of staying with the Joneses. As long as this productivity is strong enough, specific KUJ inclinations shape agents utility such that they strictly prefer to cut their educational expenditures to zero in order to keep up with the reference group. Finally, agents at higher level of income invest in both educational expenditures and social participation good; however, given the non-homotheticity of the preference, the richer part of the distribution keeps on spending larger shares of its budget towards the social participation good, amplifying the size of the reference standard.

Due to the endogenization of the reference standard with the average level of consumption of the social participation good, the individual demand for participation will depend not only on one own income, but also on the mean and the distribution of incomes. The proposed framework predicts that changes in the mean and the distribution of income across agents do affect the costs of staying with the Joneses and modify the individual's reference group. In particular, changes in the first two moments of the income distribution do affect the threshold level of income at which agents participate in the social group through two steps. First, for a fixed relative poverty line an increase in the mean and a mean-preserving reduction in the spread of the distribution generate a reduction in the mass of excluded people, the ones outside the reference group. As a result, a larger mass of individuals starts racing to stay with the Joneses by spending an increasing share of its budget in the consumption of the social participation good. Due to the consequent increase in the reference standard, and hence in the relative poverty line, this process causes the emergence of a renewed mass of excluded individuals such that, in the long-run, development traps slowly emerge.

The transition of incomes within each generation is governed by dynamical systems that qualitatively change across economic regimes as a function of the mean and the distribution of incomes. I consider different configurations of conditional dynamical systems (Galor and Weil, 2000) and I distinguish two regimes depending on whether the mean income is higher or lower of a threshold. The first regime is, further, described by two stages depending on the level of the relative poverty line. In particular, the first and the second stages are identified, respectively, by a low and a middle level of the reference standard and hence of the costs for joining the reference group. Due to the low costs of social inclusion, the dynamical system of the first stage is characterized by a unique and globally stable steady state which is the unique basin of attraction for all the dynasties; those who are external to the reference group and

share their budget toward both the goods, those in the middle range of income, who completely drop investment in education to join the reference group, and the richer ones who buy both education and social participation over its benchmark level. Throughout the convergence towards the steady state, increases in mean incomes shift the relative poverty line due to the increases in the reference standard and drive the onset of the second stage characterized by the emergence of development traps. This stage involves an equilibrium in which all the dynasties have positive expenditures in education, but they are perfectly segmented into two groups; those who stay with the Joneses and those who do not. Further increases in mean incomes may lead to the second regime, which is not qualitatively different from the second stage of the first regime. The difference relies on the fact that in the second stage of the first regime, a set of middle income dynasties gives completely up the investment in education to join the reference group. In the second regime, all the families never choose to eliminate the investment in education, since the costs of social inclusion are too high. The intuition for this result is that over time, the increasing costs of social inclusion shape the transition from a Solovian-type stage to a development traps regime since agents need to sacrifice relatively more resources in order to keep up with the reference group. Due to the production benefits accruing by staying with the neighbor, there will be dynasties penalized from being excluded from the reference group and hence trapped in a low stable equilibrium.

The rest of the paper is organized as follows. Section 2 remarks the contributions of the paper to the literature. Section 3 presents the model and Section 4 analyzes the effects of the endogenization of the reference group. Section 5 discusses the dynamical system. The last section concludes.

2   Related literature

The modification proposed to the standard KUJ setting rests on two recently empirical findings; (i) the existence of concerns for relative position can be connected to productivity benefits, and (ii) KUJ concerns affect the preferences of the individuals across the distribution with a non-linear strength. Following the original ideas of Veblen (1899) and the subsequent influential contribution of Duesenberry (1949) that positionality matters for the individual preferences, relevant pieces of empirical evidence show that agents' utility does not only depend on one own consumption but also on one's relative position in the society[3]. Two relevant issues arise with regard to whether concerns for relative standing affect homogeneously the agents across the distribution and, accordingly, to how to model a specific type of concern. Banerjee and Duflo (2007) emphasize that also the very poor expend relevant shares of their

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[3]The existence and the relevance of positional concerns have been hugely tested through empirical analysis based on data from surveys on self-reported happiness, as a proxy for the utility, as well as through survey experiments. The general findings report that happiness is significantly and negatively affected by relative income and consumption levels (Card *et al.*, 2011; Clark and Oswald, 1996; Clark and Senik, 2010; Ferrer-í-Carbonell, 2005; Luttmer, 2005). Survey experiments have also been used to determine the degree of positionality of income and consumption (Alpizar *et al.*, 2005; Solnick and Hemenway, 2005); these studies suggest that people have positional concerns both with respect to income and consumption, but that they are stronger toward the consumption of more visible goods (cars, ceremonies, clothing).

budget towards goods often identified as positional such as festivals and ceremonies, or more broadly social participation goods. Notably, participation in these social activities does not appear to be driven, or at least not only, by pure pleasure nor by an altruistic behavior, but mostly by productive goals[4]. Social participation allows the creation of social networks, which spread productive benefits that are conducive of absolute gains[5]. Even though spurs towards social participation and relative position concerns are found to be relevant also at lower levels of income, the pressure to join specific reference groups and the subsequent effects on the individual utility have been estimated to be highly non-linear (Ravallion and Lokshin, 2010). In a happiness-relative income regression, Dynan and Ravina (2007) estimate that not only relative concerns are significantly and negatively correlated with individual happiness, but also, and more importantly, that they become salient only when a person reaches a threshold level of income, implying that concerns for relative position "primarily affect people who have an above-average income but are not extremely rich". Looking at the actual households consumption spendings, Ravina (2007) reports that specific keeping up with the Joneses effects are typical of higher income households. More recently, Heffetz (2011) complements these findings by developing a survey-based index of socio-cultural visibility of consumer expenditures. The author estimates a strikingly non-linear effect across the quintiles of the income distribution since no correlation between the index and the income elasticities of several categories of goods is found at the bottom two quintiles, while it results relevant for the top three quintiles. Particularly for the more visible, social participation, goods, a strong correlation is found only at the higher quintiles.

On a side, this paper contributes to the theoretical literature[6], providing a novel setting to study the effects of KUJ concerns and assuming that they are active due to the productive gains accruing from the social participation in one own reference group. I depart from the literature by posing that social participation over the reference standard generates an extra utility premium that introduces a non-convexity in the individuals' preferences. As suggested originally by Lewis and Ulph (1988), this specification is close to the one used also by Barnett *et al.* (2010), who are interested in the possibility that agents choose to drop out from the rat race to join the reference group. While the underlying assumption in Barnett *et al.* (2010), as common in the literature, connotes the KUJ attitudes as hard-wired in the individual preferences, the instrumental approach proposed in this paper allows to capture distinct reactions of the agents depending on the productivity of staying with the Joneses parameter. As long as this productivity is strong enough, KUJ concerns come into effect and agents are incentivized

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[4]Rao (2001a,b) document that the poor participate and organize public ceremonies to join social networks that may help them to cope with poverty. Cole *et al.* (1992) propose that investments in the social participation goods may affect the result on the marriage market determining the possibility of escaping also absolute poverty (see also Bloch *et al.*, 2004).

[5]Calvó-Armengol *et al.* (2009) supply a peer-effects model, where the individual and equilibrium outcomes depend on the network centrality measure of the agent; they estimate that a standard deviation increase in the centrality of the individual within the network increases the pupil school outcome by more than 7% of one standard deviation.

[6]Several specifications have been advanced, discriminating broadly between concerns for one's relative level of either consumption, income or wealth (Abel, 1990; Dupor and Liu, 2003; Galí, 1994), or yet concerns for one own rank on the distribution (Frank, 1985; Hopkins and Kornienko, 2004).

in staying with the Joneses since this interaction induces productive advantages as well. Otherwise, if the productivity is low enough, the running away from the Joneses (RAJ) attitude (Dupor and Liu, 2003; Kawamoto, 2009), which indicates that individuals care of deviating from the mean (i.e. common) behavior of the society, becomes active. Further, I employ a dynamic framework and I find that changes in the first two moments of the distribution affect the threshold level of income at which agents participate in the social group; hence, over time, changes in the mean and the distribution of incomes qualitatively modify the reference group by influencing the costs of social inclusion.

This result implies, on the other side, that the proposed theory provides insights also to three strands of the macroeconomic literature on economic growth. First, this paper contributes to the very tiny literature on poverty traps in models with status concerns. Surprisingly enough, few studies investigate the link between KUJ inclinations and development traps. Most of the works focus on how the optimality of the growth rates and the speed of convergence toward the (unique) steady state are affected by the introduction of interdependent preferences in otherwise standard neoclassic growth models (Alvarez-Cuadrado *et al.*, 2004; Carroll *et al.*, 1997; Liu and Turnovsky, 2005). Others remark an incentive effect that leads to either a catching up of the poorer agents with the richer ones (Long and Shimomura, 2004) or to a positive effect on the unique growth rate of steady state (Corneo and Jeanne, 2001; Futagami and Shibata, 1998), as the rat race would induce the individuals to choose less leisure and more work[7]. The unique exception, to my knowledge, is provided by Moav and Neeman (2010), whose analysis is very close in the spirit to the theory presented in this paper[8]. The authors advance that conspicuous consumption (Veblen, 1899), used as a signal for unobserved income, drives to poverty traps due to the incentives of the poor to separate themselves from the extremely poor in order to enjoy the status of the high income class. I deviate from their study, introducing an instrumental setting able to highlight a different channel that leads to the emergence of development traps; namely, the relevance of the variation, over time, in the costs of joining the reference group induced by the particular discontinuity of the preferences. At this regard, my paper is also related to the earliest class of models on poverty traps that emphasized capital market imperfections and indivisibilities in production as drivers of long-run multiple stable equilibria (among many Galor and Zeira, 1993; Banerjee and Newman, 1993); here I replace the non-convexities in production with those in the preferences (Mani, 2001; Moav, 2002), presenting a mechanism that, as argued in the introductory example, does not necessarily require indivisibilities in the characteristics of the social participation good to generate the non-homotheticity. Finally, and more notably, the last element of departure from the analysis of Moav and Neeman (2010) is that the onset of development traps is neither an instantaneous event nor it depends on some specific parameter configuration, but it is a

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[7]See Hopkins and Kornienko (2006) for a contrary argument.

[8]Kawamoto (2009) only tangentially refers to the relation between KUJ concerns and poverty traps, as their main interest is on the effects for the dynamics of inequality (at this regard, see also Cole *et al.*, 1992; García-Peñalosa and Turnovsky, 2008).

by-product of the long-run evolution of the economy as the dynamical systems shift from a Solovian-type stage to a development traps regime since agents need to sacrifice relatively more resources in order to keep up with the reference group[9]. These properties of the dynamical process entail also a connection with the literature on Unified Growth Theory (Galor 2010, and reference therein). Galor (2010) emphasizes that deep factors that may have affected, through the development process, either the rate of technological progress or the accumulation and the composition of human capital provide a comparative perspective for the study of the modern divergences in per-capita income across the world. The proposed theory may be viewed as a grain of contribution into the study of the factors that, over time, may have induced a slow emergence of multiple economic regimes through their effects on the incentives and the returns of the human capital accumulation. Along this line, the predictions of the model are consistent with the empirical evidence on the existence of multiple growth regimes and non-linearities in the evolution of the growth rates (Bloom *et al.*, 2003; Fiaschi and Lavezzi, 2003) as well as with more recent empirical findings on the pattern of relative poverty rates[10].

3 The model

3.1 Structure

A continuum of heterogeneous families, indexed by i and each composed of a parent and a child, is modeled in an overlapping generation economy in which total population is constant over time. Agents are differentiated by their income endowments that are determined by previous generations and distributed according to a distribution function $G_t(y_t^i)$ defined on the support $[0, \bar{y}]$, with density $g_t(y_t^i)$, mean income $(\bar{y}_t)$ and standard deviation (σ_t). Individuals live two periods, dying at the end of the second one. In the first period of their life (childhood) children obtain education, financed by their parents. In the second period (adulthood), parents supply their efficiency units of labor, receive a wage and choose how to split their budget constraint over two goods; education for their children (e) and a social participation good (z).

3.2 Production

The production of the single homogeneous good is linear in human capital[11]:

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[9]See also Artige *et al.* (2004) that analyze how consumption habits have affected the pattern of reversals of leadership in the historical process.

[10]Ravallion and Chen (2011) and international reports (OECD, 2008) show that, in the last decades, relative poverty increased both in the developing and in the OECD countries. Ravallion and Chen (2011) show that, for a sample of 116 developing countries across the world in the interval between 1981 and 2005, the number of relatively poor rose from 2.3 to 2.6 billion, while that of absolute poor shrank. Similarly, between the mid-1980s and the mid-2000s the number of the relatively poor increased in two-thirds of the 24 OECD countries (OECD, 2008); the cumulative increase was around 1.2 points of the headcount index, corresponding to an increase of 13% of relatively poor people.

[11]Alternatively, physical capital could be introduced by assuming a small open economy with perfect capital mobility; in this environment, the rate of interest, and hence the dynamics of the physical capital, would be internationally fixed without affecting the results.

$$Y_t = H_t = \int_{i \in I} h_t^i g_t \left(h_t^i \right) dh_t^i \tag{1}$$

where H_t is the aggregate stock of human capital at time t, with $\int_{i \in I} g_t \left(h_t^i \right) dh_t^i = 1$. In each period, adults inelastically supply their efficiency units of labor receiving a wage, equal to one, so that their disposable income is

$$y_t^i = h_t^i \tag{2}$$

3.3 Individuals

At time t, the preferences of the parents (born at $t-1$) are defined over their second period consumption of the social participation good and the human capital of their children[12]:

$$u_t^i \left(z_t^i, e_t^i; \theta \right) = \ln \left(z_t^i + \tilde{\theta} \left(z_t^i \right) \right) + \gamma \ln h_{t+1}^i \tag{3}$$

with

$$\tilde{\theta} \left(z_t^i \right) = \begin{cases} 0 & \text{if } z_t^i < \kappa \\ \theta & \text{if } z_t^i \geq \kappa \end{cases} \tag{4}$$

with $\theta > 0$, and $\gamma > 0$ is the degree of altruism of the parents[13]. Consumption of the participation good (z) over the reference standard κ generates an utility premium (θ) that captures the satisfaction for staying with the Joneses as it represents the strength of the relative concerns for relative position. This feature introduces non-homotheticity only in the preferences of the agents with higher incomes. It reveals that the closer the individuals are to a relative poverty line (i.e. an income threshold) to be endogenously determined, the higher are their incentives in changing behavior towards the purchase of this participation good. It is noteworthy to stress that, notwithstanding the existence of (innate) relative concerns in the individual preferences (θ), KUJ attitudes are not yet defined since they stem from the productive advantages associated with the accumulation of the children's human capital $\left(h_{t+1}^i \right)$.

Parents contribute to the accumulation of children's human capital, and hence their income, through two channels (Galor and Tsiddon, 1997): directly, by means of the educational expenditures (e_t^i) and indirectly, by bequeathing them an informational advantage that they obtain consuming the participation good over the threshold (κ). This latter effect is described by the following production function

$$\psi \left(\tilde{\theta} \left(z_t^i \right) \right) = \eta \tilde{\theta} \left(z_t^i \right) \tag{5}$$

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[12]Introducing a normal good (c_t) to be consumed over some fixed subsistence level $(\tilde{c})$ would not change the results, but complicates only the analysis.

[13]Parents care of the well-being of their children according to a *joy of giving* motive (Andreoni, 1989).

where $\eta > 0$ is the productivity of staying with the Joneses. Finally, the human capital technology is

$$h^i_{t+1} = h\left(e^i_t, z^i_t\right) = \left(\psi\left(\tilde{\theta}\left(z^i_t\right)\right) + e^i_t\right)^{\beta} \tag{6}$$

with $0 < \beta < 1$. The children's level of human capital is an increasing, concave function of educational expenditures. Further, $h\left(0, z^i_t\right) = (\eta\theta)^{\beta}$, and $\lim\limits_{e^i_t \to 0} h^{i'}_{t+1}\left(e^i_t\right) > 0$ only for agents at higher levels of income, for whom $z^i_t \geq \kappa$ holds; the richer have a comparative advantage with the respect to the poorer in the accumulation of human capital due to an informational benefit coming from the social participation. As it will be clear from the optimization, this production gain shapes the incentive to consume the participation good and induces the concern for relative position to assume the specific form of keeping up with the Joneses attitude.

Finally, from (2) individuals in their second period of life (parenthood) face the following budget constraint

$$z^i_t + e^i_t \leq y^i_t \tag{7}$$

3.4   Optimization

In each period t, each adult individual i chooses z and e that maximize utility in (3), given an endowment of human capital h^i_t, determined by previous generations, subject to (4), (5), (6), and (7). Individually, for each of the two possible specifications suggested by (4), utility is continuous, strictly increasing and concave. Hence, the problem can be solved analyzing, independently, the two conditional optimization problems; the one for not participating (np, $\theta = 0$) and the one for participating (p, $\theta > 0$) in the reference group. The optimal solutions will derive from the comparison of the conditional indirect utility functions, associated with the two problems.

By substituting (6) in (3), not participating generations, who are left out from the reference group, solve the problem of choosing z and e such that

$$\left\{z^i_t, e^i_t; \theta = 0\right\} = \arg\max\left\{\ln z^i_t + \alpha \ln e^i_t\right\} \tag{8}$$

subject to (7) and

$$\left(z^i_t, e^i_t\right) \geq 0,\ z^i_t < \kappa \tag{9}$$

with $\alpha \equiv \gamma\beta$. The first order conditions are

$$z^{i,np}_t = \frac{y^i_t}{1 + \alpha}, \quad e^{i,np}_t = \frac{\alpha}{1 + \alpha} y^i_t \tag{10}$$

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which are valid solutions as long as $z_t^i < \kappa$, which implies that $y_t^i < \hat{y} \equiv \kappa (1 + \alpha)$; agents do split their budget proportionally across the two goods, due to the homotheticity of the preference in this range. Let define as $v^{np} \left(y_t^i; \theta = 0 \right)$ the conditional indirect utility function associated to these solutions.

Correspondingly, participating generations, the Joneses, solve a similar problem choosing z and e such that

$$\left\{ z_t^i, e_t^i; \theta > 0 \right\} = \arg\max \left\{ \ln \left(z_t^i + \theta \right) + \alpha \ln \left(\eta \theta + e_t^i \right) \right\} \tag{11}$$

subject to (7) and

$$\left(z_t^i, e_t^i \right) \geq 0, \; z_t^i \geq \kappa \tag{12}$$

The optimal solutions are defined by the first order conditions as

$$z_t^{i,p} = \frac{y_t^i}{1 + \alpha} + \frac{\theta (\eta - \alpha)}{1 + \alpha}, \quad e_t^{i,p} = \frac{\alpha}{1 + \alpha} y_t^i - \frac{\theta (\eta - \alpha)}{1 + \alpha} \tag{13}$$

which are valid solutions as long as $z_t^i \geq \kappa$, which implies that

$$y_t^i \geq \tilde{y} \equiv \kappa (1 + \alpha) - \theta (\eta - \alpha) \tag{14}$$

The conditional indirect utility function, associated to this problem, is defined by $v^p \left(y_t^i; \theta > 0 \right)$.

Lemma 1 illustrates the first results according to which it is the production advantage deriving from staying with the Joneses that shapes the concerns for relative position in the specific form of KUJ inclinations (point 3a).

Lemma 1 (Preference properties). *For each $z \geq \kappa : \theta > 0$*

1. *Hierarchy: $\varepsilon_{u_z} > \varepsilon_{u_e}$.*

2. *Relative satiation: $\varepsilon'_{u_e} (e) > \varepsilon'_{u_z} (z)$, as long as $\eta > \alpha^2$.*

3. *KUJ:*

   (a) $\dfrac{d\xi}{d\theta} > 0$, *as long as $\eta > \alpha$.*

   (b) $\dfrac{d\xi}{d\eta} > 0$, $\dfrac{d\xi}{d\theta d\eta} > 0$.

   where $\varepsilon_{u_j} \equiv -\frac{u''(.)j}{u'(.)}$ is the elasticity of the marginal utility, $\varepsilon'_{u_j} (j) \equiv \frac{\partial \varepsilon_{u_j}}{\partial j}$, with $j = z, e$, and $\xi \equiv u_z / u_e$ the marginal rate of substitution between the two goods[14].

The first two properties ensure that higher priority is devoted to the participation good, when a threshold level of income is reached. While property 1 implies that participation is preferred to education

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[14] All the proofs are collected in the Appendix, if not otherwise reported.

in a neighborhood of the reference standard, property 2 ensures that the former will be hierarchically preferred to the latter also as incomes increase, since education satiates relatively earlier than participation. Property 3, finally, illustrates the condition for the existence of the incentives to keep up with the reference group. Notably, KUJ concerns (3a) become active if and only if the productivity advantage of staying with the Joneses is strong enough; that is, if and only if $\eta > \alpha$. The definition of KUJ, as $d\xi/d\theta > 0$ (3a), is close to the one proposed by Dupor and Liu (2003)[15] and indicates that the cost of giving up an amount of participation for education is increasing in the strength of the relative concern (θ). This cost is, further, increasing in the production gains such that the higher are these latter, the stronger are the KUJ concerns (3b). As explained above, the difference with Dupor and Liu (2003) is that here KUJ concerns are present only due to the production benefits accruing from the interaction with the reference group[16]. Throughout the paper I maintain that KUJ are present.

Assumption A1. $\eta > \alpha$.

As long as agents have concerns for keeping up with the Joneses, they are induced in purchasing higher amounts of the participation good, even at the cost of reducing the expenditures in education.

Remark 1. *If A1 is satisfied, KUJ effect implies $z_t^{i,p} > z_t^{i,np}$ and $e_t^{i,p} < e_t^{i,np}$.*

In order to highlight this point, notice, first, that the solutions for education of participating agents take the following configuration

$$
e_t^{i,p} \begin{cases} = 0 \text{ if } & y_t^i \leq \tilde{\tilde{y}} \\ > 0 \text{ if } & y_t^i > \tilde{\tilde{y}} \end{cases} \tag{15}
$$

with

$$
\tilde{\tilde{y}} \equiv \frac{\theta(\eta - \alpha)}{\alpha} \tag{16}
$$

In order to join the reference group, it is well likely that expenditures in education need to be reduced to zero. In particular, the threshold level of income at which the individuals join the Joneses (14) is lower than that at which agents do not participate, $\hat{y}$. This implies that there exists an interval of incomes, $y_t^i \in [\tilde{y}, \hat{y}]$, in which both the solutions - participating and not - could be potentially valid optima. Hence,

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[15]Dupor and Liu (2003) define KUJ as the case with a positive first derivative of the marginal rate of substitution with respect to the reference standard (κ). Instead, it is here defined in terms of the variation with respect to the strength of the concern for relative position (θ) since the utility is not differentiable in κ, while it is in the interval defined in the lemma. Nonetheless, the underlying idea keeps. In particular, if $\eta > \alpha$ KUJ concerns arise, while if $\eta < \alpha$ do RAJ. Throughout the paper, only the effects of the former case (i.e. KUJ) are analyzed as the effects of RAJ will be identifiable correspondingly. Finally, notice that one of the conditions in points (2) and (3a) is redundant; if $0 < \alpha < 1$, condition in (3a) implies that in point (2), and the reverse if $\alpha > 1$.

[16]Alternatively, the utility function in (3) could have been defined as $u_{i,t}(z_{i,t}, e_{i,t}; \theta) = \ln z_{i,t} + \gamma \ln h_{i,t+1}$, with $h_{i,t+1}$ and $\tilde{\theta}(z_{i,t})$ still defined as in (4), (5) and (6). None of the results would change. The specification proposed in text is more general and it allows to distinguish different behaviors of the individuals depending on the productivity parameter.

the optimal solutions and the equilibrium derive from the analysis of the indirect utility functions such as described in Lemma 2 and Proposition 1.

Lemma 2. *For each $\theta, \eta > 0$*

1. $v^p\left(y_t^i\right) > v^{np}\left(y_t^i\right)$

2. $v^p\left(y_t^i\right)|_{e>0} > v^p\left(y_t^i\right)|_{e=0}$

Lemma 2 describes the unconstrained problems and establishes that participating is always better than not participating, conditional on having positive amounts of education (point 1); further, and more straightforwardly, it shows that choosing to participate with positive expenditures of education is always better than participating with $e_t^i = 0$ (point 2). The relevance and the influence of KUJ concerns are, finally, highlighted in the following Proposition

Proposition 1. $\forall z_t^i, e_t^i \geqslant 0, \; z_t^i < \kappa : \theta = 0, \; \tilde{y} < \tilde{\tilde{y}}$

$$v^p\left(\tilde{y}\right) \equiv u\left(\kappa, 0; \theta\right) > u\left(z_t^{i,np}, e_t^{i,np}; 0\right) \equiv v^{np}\left(\tilde{y}\right)$$

Proposition 1 shows that the benefits from participation are so huge that the agent is well willing to give completely up education in order to buy the participation good. In particular, as long as the productivity advantage of staying with the Joneses is high enough, agents are well favorable to cut education to zero in order to keep up with the reference group, instead of sharing the same income level across both the goods but not being able to enjoy the extra benefits induced by the participation. Hence as y_t^i reaches the threshold $\tilde{y} < \hat{y}$, agents choose to participate, even though this should imply a complete drop in education. This is due to the fact that participating does produce a boost in the overall utility that results in a jump in the indirect utility function (Fig. 1). Notice that, for given positive strength of the concerns for relative position ($\theta > 0$), the higher is the productivity advantage (η), the higher is the amount of consumption devoted to participation, the lower is the income threshold ($\tilde{y}$) at which agents decide to give up education to access the benefits of keeping up with the reference group, and the higher is the threshold level of income ($\tilde{\tilde{y}}$) at which agents restore positive amount of education. These outcomes would be reinforced by stronger concerns for relative position - higher θ, and they are explained by the productive gains accruing by keeping up with the reference group; the higher these productivity benefits, the stronger the KUJ concerns, the higher the relative cost of choosing education instead of the participation good[17]. The final equilibrium is described by the following optimal solutions

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[17]These results definitely differentiate my approach from that of Barnett *et al.* (2010); not only an instrumental framework is employed to explain the emergence of KUJ, but, while the former authors study the possibility of dropping out from the "race" without KUJ, here agents have stronger incentives in KUJ, which induces them to buy the participation good earlier than otherwise would they have done (i.e. $\tilde{y} < \hat{y}$).

Figure 1: Equilibrium

(a) Income path

(b) Indirect utility functions
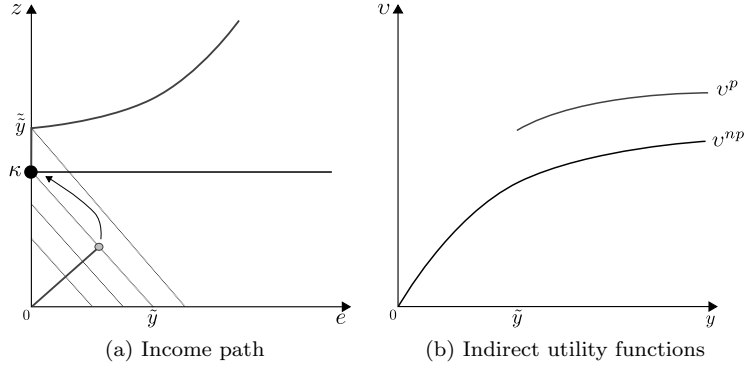
$$
\left\{ z_t^i, e_t^i \right\} =
\begin{cases}
z_t^{i,np}, e_t^{i,np} & y_t^i < \tilde{y} \\
z_t^{i,p}, 0 & y_t^i \in \left[\tilde{y}, \tilde{\tilde{y}} \right] \\
z_t^{i,p}, e_t^{i,p} & y_t^i > \tilde{\tilde{y}}
\end{cases}
\tag{17}
$$

The solutions in (17) match the stylized facts described in Section 2.

For low level of income, $y_t^i < \tilde{y}$, the homotheticity of the preferences implies that agents share their budget in fixed proportion between both goods. As incomes increase enough ($y_t^i = \tilde{y}$), KUJ concerns come into effect since the reference group is source of productive benefits; these benefits are so relevant that agents are well willing to give completely up the consumption of the other goods (i.e. education) to keep up with the reference good. Finally, at higher level of income ($y_t^i = \tilde{\tilde{y}}$), the individuals consume both participation and education (Fig. 1); moreover, the *local* non-homotheticity of the preference in this higher range of income implies that, as Lemma 1 describes, higher shares of the budget will be devoted to the participation good in order to keep on staying with the reference group (Fig. 1, panel 1a). This pattern produces the sharp discontinuity in the preference, which is described by the jump in the indirect utility function at the threshold $\tilde{y}$ (Fig. 1, panel 1b).

4   The reference group

Until this point the reference standard (κ) has been treated as fixed and exogenous. From now on, I define it as the average level of consumption of the participation good.

Definition 1 (Reference standard). *Let* $\kappa \equiv \bar{z}_t = \int_0^{\tilde{y}} z_t^{i,np} dG\left(y_t^i\right) + \int_{\tilde{y}}^{\bar{\bar{y}}} z_t^{i,p} dG\left(y_t^i\right)$

Using (10) and (13),

$$
\bar{z}_t = \int\limits_0^{\tilde{y}_t} \frac{y_t^i}{1+\alpha} dG_t\left(y_t^i\right) + \int\limits_{\tilde{y}_t}^{\bar{\bar{y}}} \frac{y_t^i}{1+\alpha} + \frac{\theta\left(\eta - \alpha\right)}{1+\alpha} dG_t\left(y_t^i\right)
$$

13

$$\bar{z}_t = \frac{\bar{y}_t}{1+\alpha} + \frac{\theta(\eta-\alpha)}{1+\alpha}(1 - G_t(\tilde{y}_t)) \qquad (18)$$

where $G_t(\tilde{y}_t)$ is the proportion of the population with an income lower than $\tilde{y}_t$; that is, the mass of people outside the reference group. If agents have no innate concern for their relative position ($\theta = 0$), the mean level of consumption of the participation good is independent from the distribution and it corresponds, as in the standard case, to a percentage of the mean income of the economy. Whenever concerns for relative standing are present ($\theta > 0$), the productivity of staying with the Joneses drives the emergence of either KUJ or RAJ concerns. Under assumption A1, KUJ concerns induce an overconsumption of the participation good, and its average level is higher than that would be in the case of no concern ($\theta = 0$), due to the race among agents to gain a better relative positions[18]. Further, the higher (lower) is $G_t(\tilde{y}_t)$, the lower (higher) is $\bar{z}_t$ since the higher (lower) is the proportion of not participating individuals, the lower (higher) is the level of consumption of the participation good.

Changes in the reference standard do affect the participation in the reference group due to its mapping into the threshold level of income, the relative poverty line, $\tilde{y}_t$; substituting (18) in (14), it follows that

$$\tilde{y}_t = \bar{y}_t - \theta(\eta-\alpha)G_t(\tilde{y}_t) \qquad (19)$$

Proposition 2. *Under A1, the relative poverty line in* (19) *exists and it is unique.*

The relative poverty line is, now, time dependent and it is characterized by the mean and the distribution of incomes. If there are no concerns ($\theta = 0$), the poverty line equals simply the mean income of the economy and it does not depend on the income distribution, while it is lower than the mean income under the KUJ hypothesis. An interpretation for why it is lower than the mean income and the reference standard (18) is that participation is so beneficial that KUJ effects induce agents to buy the reference standard well earlier than when would they have, otherwise, started to (i.e. $\hat{y}$). Further, the higher the strength of the concerns and the stronger the productivity advantage of staying with the Joneses, the lower the relative poverty line. On a side, higher θ or η or both, by producing an increase in the reference standard, suggest that more people do not purchase the participation good, which brings forth a reduction in the poverty line (Fig. 9b); on the other side, the same effect induces an increase in the incentives to participate in the reference group such that agents start buying the participation good at a lower level of the income threshold.

As it follows from (19), changes in the reference standard and in the relative poverty line are accounted for by changes in the mean income and the spread of the distribution via their impact on the mass of the agents excluded from the reference group ($G_t(\tilde{y}_t)$). In particular, two effects are analyzed; an increase

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[18]If $\eta < \alpha$, RAJ concerns imply that the average consumption of the participation good is lower than that it would be in both the case of no concern and of KUJ concern, due to the propensity of richer agents to isolate themselves from the mass consumption.

in the mean income not affecting the spread of the distribution such that the resulting distribution first order stochastically dominates the original one, and a mean preserving increasing spread such that the original distribution second order stochastically dominates the resulting distribution[19]. Formally, the first effect deals with an increase of the mean income between time $t = 0$ and $t = 1$ ($\bar{y}_1 > \bar{y}_0$) that does not affect the standard deviation such that $G_1\left(y^i\right) < G_0\left(y^i\right)$. The second effect is described by an increase in the standard deviation[20] ($\sigma_1 > \sigma_0$) that not affects the mean income such that $G_1\left(y^i\right)$ is a mean preserving spread of $G_0\left(y^i\right)$, namely that $\int_0^y G_0\left(s\right)ds \leq \int_0^y G_1\left(s\right)ds$. The results are collected in the following Proposition and Corollary

Proposition 3. *If A1 is satisfied,*

1. *For given standard deviation, an increase in mean income produces an increase in the relative poverty line*

$$\frac{\partial \tilde{y}_t}{\partial \bar{y}_t} = \frac{1 - \theta\left(\eta - \alpha\right)G_{\bar{y}_t}\left(\tilde{y}_t\right)}{1 + \theta\left(\eta - \alpha\right)g_t\left(\tilde{y}_t\right)} > 0 \tag{20}$$

2. *An increase in the level of inequality, a mean preserving increasing spread, implies a decrease in the relative poverty line*

$$\frac{\partial \tilde{y}_t}{\partial \sigma_t} = -\frac{\theta\left(\eta - \alpha\right)G_{\sigma_t}\left(\tilde{y}_t; \sigma_t\right)}{1 + \theta\left(\eta - \alpha\right)g_t\left(\tilde{y}_t; \sigma_t\right)} < 0 \tag{21}$$

Corollary 1. *If A1 is satisfied,*

1. *An increase in mean income produces an increase in the reference standard*

$$\frac{\partial \bar{z}_t}{\partial \bar{y}_t} = \frac{1}{1 + \alpha} - \frac{\theta\left(\eta - \alpha\right)}{1 + \alpha}\left[g_t\left(\tilde{y}_t\right)\frac{\partial \tilde{y}_t}{\partial \bar{y}_t} + G_{\bar{y}_t}\left(\tilde{y}_t\right)\right] > 0 \tag{22}$$

2. *An increase in the level of inequality, a mean preserving increasing spread, implies a decrease in the reference standard*

$$\frac{\partial \bar{z}_t}{\partial \sigma_t} = -\frac{\theta\left(\eta - \alpha\right)}{1 + \alpha}\left[g_t\left(\tilde{y}_t\right)\frac{\partial \tilde{y}_t}{\partial \sigma_t} + G_{\sigma_t}\left(\tilde{y}_t\right)\right] < 0 \tag{23}$$

An increase in the mean income induces a rise in the relative poverty line through two channels (Fig. 11). Overall, a higher mean income implies that an increased set of richer individuals will spend more on the participation good, causing a rise in the reference standard which brings about a subsequent increase

---

[19]Throughout we consider only symmetric and unimodal distributions; due to this, a mean preserving spread implies that the two distributions (CDF) cross at the mean income (i.e. $\bar{y}_t$). It is possible to extend these arguments also to skewed distributions (Barnett *et al.*, 2010).

[20]Given $\tilde{y}_t < \bar{y}_t$, the derivative of the distribution with respect to the standard deviation is positive, $G_{\sigma_t}\left(\tilde{y}_t\right) > 0$.

Figure 2: Relative poverty line

(a) Mean income (b) Mean preserving increasing spread
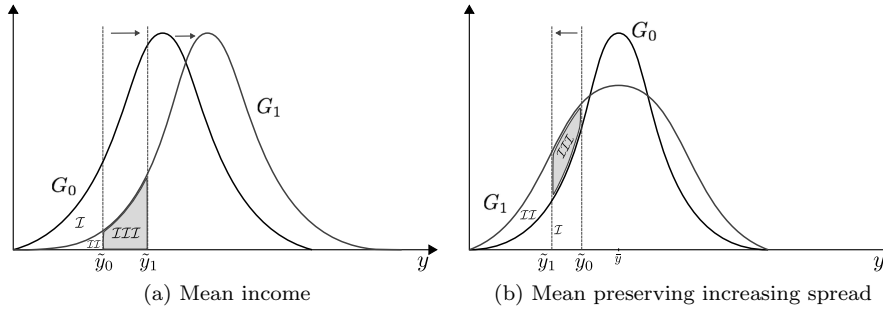
in the poverty line as well. This effect can be split in a direct force, due to the fact that the society is richer, and in a compositional effect, which passes through the reduced mass of individuals outside the reference group at the original threshold level of income ($G_1(\tilde{y}_0)$, Fig. 2a). For a fixed relative poverty line ($\tilde{y}_0$), the mass of excluded individuals under the new distribution induced by the change in the mean income (G_1) is lower than the initial one (area II instead of area $I + II$). As a result, a larger mass of individuals starts running in the race to stay with the Joneses by spending an increasing share of its budget in the consumption of the social participation good. This process causes an increase in the reference standard, the shift in the relative poverty line from $\tilde{y}_0$ to $\tilde{y}_1$ and hence the emergence of a renewed mass of excluded individuals (area III in Fig. 2a).

Further, a mean-preserving increase in the spread of the distribution generates a decrease in both the reference standard and the poverty line (Fig. 2b and 12). For a fixed relative poverty line (at $G_1(\tilde{y}_0)$), a rise in the spread increases the mass of the people not included in the reference group. Hence, the reduction in the consumption of the participation good drives the decrease in the reference standard, and consequently the reduction in the relative poverty line from $\tilde{y}_0$ to $\tilde{y}_1$ and the inclusion of a novel group of individuals in the reference group (area III in Fig. 2b).

4.1 The intensity of relative poverty

When agents have concerns for their relative standing and compare themselves with others in some reference group, those who do not have a particular good may feel relatively deprived with respect to the ones who have it. This is the original intuition behind the idea of relative deprivation[21] (Runciman, 1966). Agents who do not participate indeed suffer a discrete drop in their indirectly utility due to the fact that they are excluded from the reference group. This exclusion is salient not because agents have intrinsic, hard-wired, preferences for keeping up with the reference group, but more importantly because this exclusion causes a productive disadvantage; the ones who are left out of the reference group suffer an

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[21]Runciman (1966) supplied the following definition of relative deprivation: "[An individual] A is relatively deprived of X when: (i) he does not have X, (ii) he sees some other person or persons, which may include himself at some previous or expected time, as having X (whether or not this is or will be in fact the case), (iii) he wants X, and (iv) he sees it as feasible that he should have X " (p. 10).

utility loss specifically because staying with the reference group is a source of productive benefits, which bring about utility gains. This implies that the distributional changes not affect directly the utility of the individuals, but that they affect the indirect utility functions through their impact on the level of the reference standard and hence on the level of the relative poverty line. A very crude measure of the incurred loss in the indirect utility function, as a proxy for the intensity of relative poverty felt by an individual i, can be the difference between the indirect utility of the individual with an income $y_t^i < \tilde{y}_t$ and that the same individual would enjoy at the threshold $\tilde{y}_t$[22].

Definition 2 (Intensity of relative poverty). *For each individual i with $y_{i,t} < \tilde{y}_t$, let the intensity of relative poverty be defined by*

$$P_t^i \equiv v^p(\tilde{y}_t) - v^{np}(y_t^i) = (1+\alpha)\ln\left[\frac{\bar{y}_t + \theta(1 + \eta - (\eta - \alpha)G_t(\tilde{y}_t))}{y_t^i}\right] \qquad (24)$$

The intensity of relative poverty is an increasing function of the distance between the mean income and the income of the agent not included in the reference group. The higher the proportion of the Joneses (i.e. lower $G_t(\tilde{y}_t)$), the higher the intensity of relative poverty felt by who does not stay in the reference group. The stronger the strength of the concerns for relative position (θ) and the stronger the productivity advantage of staying with the Joneses (η), the higher the intensity of relative poverty.

Proposition 4. *An increase in mean income produces an increase in the intensity of relative poverty; $\frac{\partial P_t^i}{\partial \bar{y}_t} > 0$. A mean-preserving increase in the spread implies a reduction in the intensity of relative poverty; $\frac{\partial P_t^i}{\partial \sigma_t} < 0$.*

Proof. It follows from Proposition 3. □

Proposition 4 implies that an increase in the mean income induces an increase in the intensity of relative poverty. Correspondingly, a mean-preserving increase in the spread implies a reduction of the intensity of relative poverty since some of the agents, who had been excluded from the reference group at the initial threshold, would have instead joined it after the dynamic effect of the changes in the relative poverty line kicked in (those with an income between $\tilde{y}_0$ and $\tilde{y}_1$, Fig. 2b).

5 The dynamical system

The evolution of each dynasty is determined by the accumulation of the human capital of the children that depends on the trade-off faced by the parents between education and participation in the reference group. The transition of the incomes within each generation is governed by dynamical systems that, over time, qualitatively change as a function of the cost of staying with the Joneses $\tilde{y}_t$ and, hence, of the mean

---

[22]This is in line with a particular feature of relative deprivation: "relative deprivation means that the sense of deprivation is such as to involve a comparison with the imagined situation of some other person or group" (Runciman, 1966, p. 11)

and the distribution of incomes. At each time t, each agent makes her optimal choice, given her income y_t^i and considering as given the relative poverty line $\tilde{y}_t$; thereafter, the optimal choices of all the agents determine the poverty line of the next period. Due to the timing of this process, the state variable $\tilde{y}_t$ can be treated, in each period, as temporary exogenous. I consider different configurations of conditional dynamical systems (Galor and Weil, 2000), each of them defined for given threshold $\tilde{y}_t$ and, hence, for given distribution of income. In particular, three levels of the relative poverty line, corresponding to three different stages of the economy, are initially considered; a low level $\tilde{y}_t^l$, which can be due to either low mean income or high inequality or both, an intermediate level $\tilde{y}_t^m$, and finally a high level of the poverty line $\tilde{y}_t^h$, implied by either high mean income or low inequality or both. Subsequently, I demonstrate that, in the long-run, the individual dynamics must evolve towards a system characterized by development traps in which some dynasty remains trapped in a low stable equilibrium. Depending on the level of the poverty line, two regimes can be distinguished throughout. The first one (Regime I) is defined as the phase of the economy characterized by either low mean income or high inequality or both and such that $\tilde{y}_t \leq \tilde{\tilde{y}}$; correspondingly, the second regime (Regime II) is defined as the phase of the economy characterized by either high mean income or low inequality or both and such that $\tilde{y}_t > \tilde{\tilde{y}}$. The difference between the two regimes is explained by the fact that in the first one, given that the relative poverty line is lower than the threshold level at which agents restore the consumption of education, a mass of low-middle income agents chooses to completely drop education in order to buy the participation good at the threshold. In the second regime, the unique threshold which determines the choices of the agents is the relative poverty line; in this regime, whenever individuals keep up with the reference group consuming the participation good at the threshold, they will at positive amounts of educational expenditures.

5.1 Regime I $\left(\tilde{y}_t \leq \tilde{\tilde{y}}\right)$

The first regime is defined as the phase of the economy in which the relative poverty line is lower than the threshold level of income at which agents do restore educational expenditures such that $\tilde{y}_t \leq \tilde{\tilde{y}}$. This implies that some middle-income agents drop completely their educational expenditures in order to stay with the Joneses. This regime is, further, characterized by two stages of the economy, defined by the two thresholds $\tilde{y}_t^l$ and $\tilde{y}_t^m$. Formally, using (16) and (19), the first regime is defined by the condition

Remark 2 (Regime I). *Under A1, Regime I does exist as long as*

$$\bar{y}_t \leq \frac{\theta\left(\eta - \alpha\right)\left(1 + \alpha G_t\left(\tilde{y}_t\right)\right)}{\alpha} \tag{25}$$

This condition is verified whenever either the mean income is low enough or the inequality[23] is high enough or both. Noticeably, this regime does exist if and only if KUJ concerns are present $(\eta > \alpha)$;

---

[23]From proposition 3 and corollary 1, higher inequality implies a higher proportion of individuals under the poverty line $(G_\sigma\left(\tilde{y}_t\right) > 0)$.

further, the stronger the concerns for relative position and the higher the productivity of staying with the Joneses, the higher the probability that the economy will evolve experiencing this phase.

The equilibrium condition (17) implies that the optimal investment in education in this regime is given by:

$$
e_t^i =
\begin{cases}
\dfrac{\alpha}{1+\alpha} y_t^i & y_t^i < \tilde{y}_t \\[2ex]
0 & y_t^i \in \left[\tilde{y}_t, \tilde{\tilde{y}}\right] \\[2ex]
\dfrac{\alpha}{1+\alpha} y_t^i - \dfrac{\theta(\eta - \alpha)}{1+\alpha} & y_t^i > \tilde{\tilde{y}}
\end{cases}
\tag{26}
$$

Substituting (26) in the law of accumulation of the human capital (6), the evolution of income within a dynasty is determined by:

$$
y_{t+1}^i =
\begin{cases}
\left(\dfrac{\alpha}{1+\alpha} y_t^i\right)^\beta \equiv \phi^L\left(y_t^i\right) & y_t^i < \tilde{y}_t \\[2ex]
(\eta\theta)^\beta \equiv \phi^M\left(y_t^i\right) & y_t^i \in \left[\tilde{y}_t, \tilde{\tilde{y}}\right] \\[2ex]
\left(\dfrac{\alpha}{1+\alpha}\right)^\beta \left(y_t^i + \theta(1+\eta)\right)^\beta \equiv \phi^H\left(y_t^i\right) & y_t^i > \tilde{\tilde{y}}
\end{cases}
\tag{27}
$$

**Lemma 3.** *If Assumption A1 and (25) are satisfied, the system in (27) is characterized by the following properties:*

1. $\phi^L(0) = 0$, $\phi^M(0) = (\eta\theta)^\beta$, $\phi^H(0) = \left(\dfrac{\alpha\theta(1+\eta)}{1+\alpha}\right)^\beta > 0$

2. $\phi^{j'}\left(y_t^i\right) > 0$, $\phi^{j''}\left(y_t^i\right) < 0$, *with* $j = L, H$

3. $\lim\limits_{y_t^i \to 0} \phi^{L'}\left(y_t^i\right) = \infty$, $\lim\limits_{y_t^i \to 0} \phi^{H'}\left(y_t^i\right) > 0$

4. $\phi^M(\tilde{y}_t) > \phi^L(\tilde{y}_t)$, $\phi^M(\tilde{\tilde{y}}) = \phi^H(\tilde{\tilde{y}})$.

Lemma 3 ensures that at the relative poverty line $\tilde{y}_t$ there is a positive jump in the transitional equation, which corresponds to the jump in the indirect utility function (Fig. 1b), while there is no jump at the threshold $\tilde{\tilde{y}}$. The threshold $\tilde{\tilde{y}}$ is fixed and its extent depends on the strength of the concerns for relative position (θ) and on the productivity of staying with the Joneses (η). Two cases will be distinguished throughout; a low concerns case, described by either low η or θ or both, and a high concerns one, described by high η or θ or both. Noticeably, the differences between these two cases not qualitatively affect the transition towards a development traps equilibrium, but they will affect only the magnitude of the upper steady state equilibrium. In particular, if Assumption A1 holds, starting from a Solovian-type stage, characterized by a unique stable steady state, the economy evolves toward a development trap equilibrium with multiple stable steady states, regardless of the extent of these concerns. In order to characterize the first regime, the following Lemma determines the position of the fixed threshold $\tilde{\tilde{y}}$:

Lemma 4. *If Assumption A1 and (25) are satisfied, there exists a downward sloping locus, defined by the couple $(\overset{\circ}{\eta}, \overset{\circ}{\theta})$, such that for any couple $(\eta, \theta) < (\overset{\circ}{\eta}, \overset{\circ}{\theta})$: $\phi^M(\tilde{y}) > \tilde{y}$ (low-concerns) and for any $(\eta, \theta) > (\overset{\circ}{\eta}, \overset{\circ}{\theta})$: $\phi^M(\tilde{\tilde{y}}) < \tilde{\tilde{y}}$ (high-concerns).*

Low concerns case The first regime is defined by two stages, depending on the level of the relative poverty line; a low relative poverty line $(\tilde{y}_t^l)$ describes the stage I (Fig. 3), while an intermediate level $(\tilde{y}_t^m)$ introduces to the second stage (stage II, Fig. 4) of this regime. In the first stage, the dynamical system presents a unique steady state level of income towards which agents converge in the long-run (y_H^*). The stability of this steady state depends on two sets of properties. On a side, it depends on the properties of the steady state, which are characterized by Lemma 3; on the other side, it depends also on the stability of the relative poverty line $\tilde{y}_t^l$. It is, indeed, shown that the first stage of the first regime (Fig. 3) cannot be a sustainable equilibrium in the long-run.

Figure 3: Dynamical system: low-concern, regime I - stage I
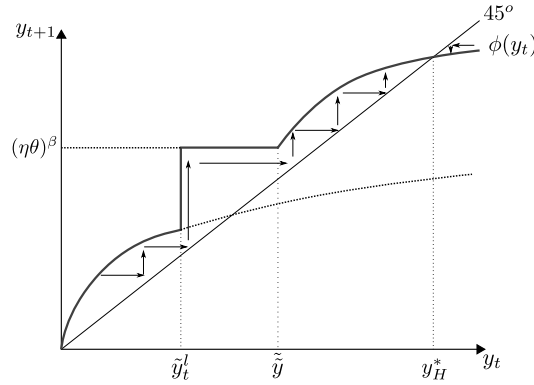
In the first stage of the first regime, Lemma 3 implies that all the dynasties, regardless of their initial level of income, converge towards the unique stable steady state (y_H^*). Throughout this convergence, the increase in mean income and the reduction in inequality cause, from Proposition 3 and Corollary 1, the increase of the relative poverty line $(\tilde{y}_t^m)$ and, as a result, the onset of the stage II of the first regime (Fig. 4). This stage is characterized by the emergence of development traps, with a dynamical system featuring three steady state equilibria, two of which globally stable $(y_L^*$ and $y_H^*)$ and the other unstable.

The following proposition establishes a powerful result, which guarantees that in the long-run the economy must converge toward an equilibrium characterized by development traps.

Proposition 5 (Emergence of development traps). *Under Assumption A1 and Lemma 3, for a given initial distribution $G_0\left(y_0^i\right)$ and an income y_0^i, there must exist a time $\tilde{t}$ at which development traps do emerge so that the evolution of the incomes within each generation is governed by a dynamical system that exhibits multiple stable equilibria.*

Proof. The proof starts by showing that the dynamical system describing the stage I (Fig. 3) cannot be

Figure 4: Dynamical system: low-concern, regime I - stage II
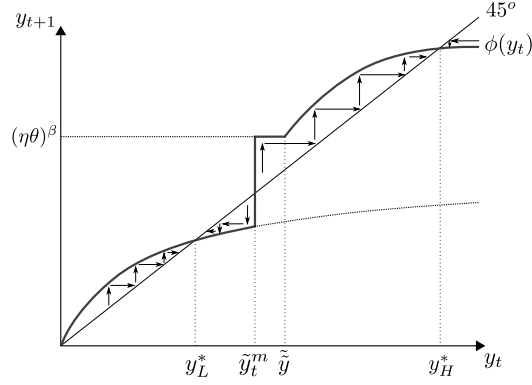
a long-run equilibrium. At this end, let consider an economy in which at time $t = 0$ KUJ concerns are present, which entails that $\theta > 0$ and $\eta > \alpha$. Assumption A1 and Proposition 2 ensure that the threshold $\tilde{y}_0$ exists, it is unique and it is defined as in (19). Suppose, further, that the system of stage I is a long-run equilibrium so that, given Lemma 3, y_H^* is a globally stable steady state. In order for any agent to complete the process of convergence toward y_H^*, it should be verified that $\phi^L(\tilde{y}_t) > \tilde{y}_t$ holds everywhere along the transition. If so, anyone does converge toward this equilibrium and the mean income at the steady state is equal to the poverty line (19), and both are equal to the steady state level of income; formally, it should be verified that $\bar{y}^* = \tilde{y}^* = y_H^*$ since, furthermore, at this point inequality drops at zero $(\sigma^* = 0)$. Notice that $\lim_{\sigma_t \to 0} G(y_t^i) = F(y_t^i)$, where $F(y_{i,t}) = \begin{cases} 0 & y_t^i < \tilde{y}_t \\ 1 & y_t^i \geq \tilde{y}_t \end{cases}$ is a discontinuous distribution defined by a unit step function, so that $G(\tilde{y}_t)$ presents a discontinuity at $\tilde{y}_t$. From Proposition 2, the existence of $\tilde{y}$ would be guaranteed *iff* $\theta(\eta - \alpha) = \bar{y}^* - \tilde{y}^* = 0$, which would imply either $\theta = 0$ or $\eta = \alpha$ or both at the steady state, contradicting the KUJ hypothesis. Conversely, Propositions 2 and 3 and Corollary 1 imply that along the transition there must exist a time $\tilde{t}$ at which $\phi(\tilde{y}) = \tilde{y}$ such that the economy enters in the stage II (Fig. 4). □

In the initial stages, when the mean income is low and inequality is high, each dynasty converges toward the unique steady state $(y_H^*$, Fig. 3) as the costs of social inclusion are low as well $(\tilde{y}_t^l)$. Throughout this transition, from Proposition 3 and Corollary 1, the increase in the mean income and the reduction in inequality involve an increase in the reference standard and in the poverty line that brings to light a renewed mass of excluded individuals and drive the onset of the development traps. At this stage, some of the (low-middle income) agents remains trapped in the lower equilibrium $(y_L^*$, Fig. 4) as for them it results too costly to participate in the reference group, while some of the (high income) agents converge to the higher equilibrium (y_H^*), so that the intensity of relative poverty does increase throughout (Proposition 4). This process entails an equilibrium in which all the dynasties have positive expenditures in education, but they are perfectly segmented into two groups; those who stay with the Joneses and

Figure 5: Dynamical system: high-concern

(a) regime I - stage I   (b) regime I - stage II
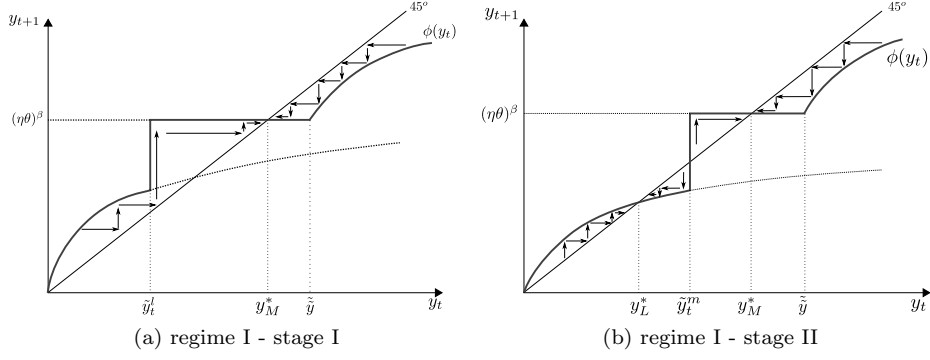
those who do not.

High concern case As long as KUJ concerns are present, the dynamical system is not qualitatively affected by the extent of the strength of the concerns for relative position (θ) and the productivity of staying with the Joneses (η). However, if the concerns are high enough so that it exists a couple $(\eta, \theta) > (\overset{\circ}{\eta}, \theta)$ that implies that $\phi^M(\tilde{\tilde{y}}) < \tilde{\tilde{y}}$, strong KUJ motives do affect the magnitude of the upper equilibrium. The dynamical system is still represented by the transition equation in (27), whose properties are defined by Lemma 3; furthermore, the following Lemma applies to this case

Lemma 5. $\phi^{H'}(y_t^i)\mid_{y_t^i = \tilde{\tilde{y}}} < 1$.

Given Lemma 3 and 5, the evolution of the economy is described in Figure 5. Proposition 5 ensures that over time the economy will evolve into the stage II of the first regime, bringing about the emergence of the development traps; at this equilibrium there are three steady states, two of which are globally stable (y_L^* and y_M^*) and the other unstable. Unlike the low concerns case, if the strength of the concerns for relative position (θ) or the productivity of staying with the Joneses (η) or both are high enough, the upper equilibrium is characterized by a level of steady state per-capita income lower than that of the low concern case ($y_M^* < y_H^*$, Fig. 4 and 5). Due to the very strong KUJ concerns, agents do, indeed, overconsume in the participation good so that the lower expenditures in education lead the richer part of the society toward an intermediate steady state. In equilibrium (Fig. 5b), the poorest agents share their budget over the two goods (participation and education) and converge to the low steady state, while the richest use their budget only in the purchase of the participation good ($y_M^* < \tilde{\tilde{y}}$). This behavior is driven by the fact that the concerns for relative position and the productivity benefits accruing from joining the reference group are so strong that richer agents expend all their budget in the consumption of the participation good; its overconsumption does prevent the investment in education and leads to a steady state level of income lower than that in the low concern case.

5.2 Regime II $(\tilde{y}_t > \tilde{\tilde{y}})$

The second regime is defined as the phase of the economy in which further increases in mean income and decreases in inequality lead the relative poverty line to rise above the threshold $\tilde{\tilde{y}}$ so that the unique threshold which determines the choices of the agents is the relative poverty line $\tilde{y}_t$. This regime does exist only for low levels of concerns for relative position (i.e. low-concern case) and it is described only by one stage. In this regime, whenever individuals keep up with the reference group consuming the participation good at the threshold, they will at positive amounts of educational expenditures.

Remark 3 (Regime II). *Under A1, Regime II does exist as long as*

$$\bar{y}_t > \frac{\theta\left(\eta - \alpha\right)\left(1 + \alpha G_t\left(\tilde{y}_t\right)\right)}{\alpha} \tag{28}$$

In this regime the optimal investment in education is determined as

$$e_t^i = \begin{cases} \dfrac{\alpha}{1+\alpha} y_t^i & y_t^i < \tilde{y}_t \\[3mm] \dfrac{\alpha}{1+\alpha} y_t^i - \dfrac{\theta\left(\eta - \alpha\right)}{1+\alpha} & y_t^i \geq \tilde{y}_t \end{cases} \tag{29}$$

so that the dynamical system is described by the transition equation

$$y_{t+1}^i = \begin{cases} \left(\dfrac{\alpha}{1+\alpha} y_t^i\right)^\beta \equiv \phi^L\left(y_t^i\right) & y_t^i < \tilde{y}_t \\[3mm] \left(\dfrac{\alpha}{1+\alpha}\right)^\beta \left(y_t^i + \theta\left(1 + \eta\right)\right)^\beta \equiv \phi^H\left(y_t^i\right) & y_t^i \geq \tilde{y}_t \end{cases} \tag{30}$$

As follows from Lemma 3, the dynamical system presents a jump at this higher threshold $(\tilde{y}_t^h)$, which perfectly discriminates the population in two groups; those who are outside the reference group converge toward the lower stable steady state y_L^*, while the Joneses approach the higher stable steady state equilibrium, y_H^* (Fig. 6).

Figure 6: Dynamical system: regime II
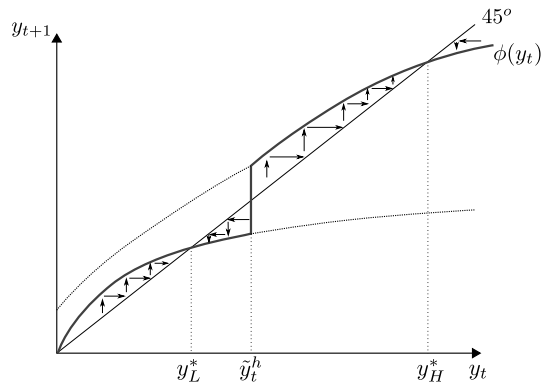
The relative poverty line cannot increase without bounds; in particular, a relative poverty line high

enough $(\tilde{y}_t^r)$ and such that $\phi^H(\tilde{y}_t) < \tilde{y}_t$ entails an equilibrium which is not sustainable in long-run. This result is formally collected in the next proposition, which shows that no reversal can arise since it is not a long-run equilibrium the convergence also of the richer agents toward a unique low steady state $(y_L^*,$ Fig. 7).

Proposition 6 (No Reversal). *Under Assumption A1 and Lemma 3, convergence toward a unique and globally low steady state is not a sustainable long-run equilibrium.*

Proof. In order for y_L^* to be the unique basin of attraction in the long-run, it must be the case that $\phi^H(\tilde{y}_t) < \tilde{y}_t$ is verified throughout the process of convergence. Suppose that such an equilibrium exists so that any agents converge to y_L^*; this implies also that $y_L^* = \bar{y}^*$. This cannot be a sustainable equilibrium since it implies that $\bar{y}_t < \tilde{y}_t$ (i.e. $\bar{y}^* < \tilde{y}_t^r$), violating Proposition 2. Further, in equilibrium inequality would drop to zero; from Proposition 5, similar arguments complete the proof, ensuring that this cannot be a long-run equilibrium. □

Figure 7: Dynamical system: no reversal
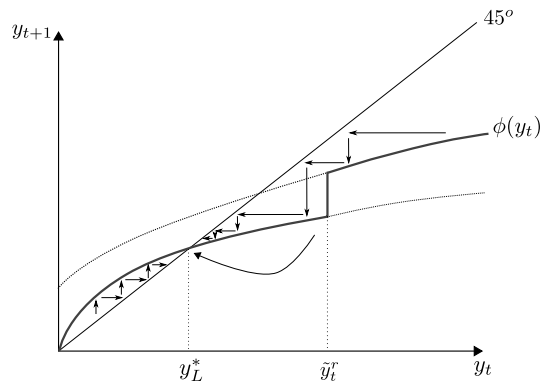
Proposition 6 guarantees that starting from an initial phase (regime I, stage I) an economy cannot show any reversal involving a convergence toward the lower equilibrium also of the richer agents. Further, it shows that if, for some reasons, an economy departs at time $t = 0$ in a case such as described in Proposition 6 and Figure 7, in the long-run the reduction in the mean income would decrease the relative poverty line such to restore the development trap equilibrium as in Figure 6. Notice, finally, that if KUJ motives are too strong (high-concern) the regime II ceases to exist. In this case, the economy would approach this regime only at a level of income for which, from Proposition 6, the equilibrium is not sustainable in the long-run (Fig. 7).

6 Concluding remarks

In this paper, I provide a modification of the standard keeping up with the Joneses setting, assuming that KUJ concerns are not hard-wired in the individuals' preferences, but that they are active due to

the productive gains accruing from the social participation in one own reference group. Concerns for relative position are shaped by two parameters; the strength of the concerns and the productivity of staying with the Joneses. As long as this productivity is strong enough, specific KUJ inclinations shape agents' utility such that a mass of low-middle income agents strictly prefer to cut their educational expenditures to zero in order to keep up with the reference group. Consuming the participation good over the reference standard, which is endogenized as the average level of consumption across the society, social participation generates, indeed, an utility premium which triggers a discrete jump in the indirect utility function. Based upon this instrumental setting, this paper highlights how changes in the costs of joining the reference group do affect the emergence of development traps. In particular, it is shown that an increase in mean income and a reduction of inequality cause an increase in the reference standard, inducing, in the long-run, the transition from a Solovian-type stage to a development traps regime as agents need to sacrifice relatively more resources in order to keep up with the reference group. Due to the production benefits accruing by staying with the neighbor, there are dynasties penalized from being excluded from the reference group and then trapped in a low stable equilibrium. The onset of development traps is neither an instantaneous event nor it depends on some specific parameter configuration, but it is a by-product of the long-run evolution of the economy as the dynamical systems qualitatively change as a function of the costs of staying with the Joneses, and, hence, of the mean and the distribution of incomes. In the initial stages, when the mean income is low and inequality is high, all the dynasties, regardless of their level of income, converge toward the unique steady state as the costs of social inclusion are low as well. Throughout this transition, the increase in the mean income and the reduction in inequality involve an increase in the reference standard and in the poverty line that brings to light a renewed mass of excluded individuals and drive the onset of the development traps.

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Appendix: Proofs

Lemma 1

1. Hierarchy: $\varepsilon_{u_z} \equiv \frac{z}{z+\theta} > \frac{e}{e+\eta\theta} \equiv \varepsilon_{u_e}$ as long as $\eta > 1$.

2. Relative satiation: $\varepsilon'_{u_e}(e) \equiv \frac{\partial \varepsilon_{u_e}}{\partial e} = \frac{\eta\theta}{(\eta\theta+e)^2}$, $\varepsilon'_{u_z}(z) \equiv \frac{\partial \varepsilon_{u_z}}{\partial z} = \frac{\theta}{(\theta+z)^2}$. $\varepsilon'_{u_e}(e) > \varepsilon'_{u_z}(z) \Rightarrow \left(\frac{z+\theta}{e+\eta\theta}\right)^2 > \frac{1}{\eta} \Rightarrow \left(\frac{z+\theta}{e+\eta\theta}\right) > \frac{1}{\eta^{1/2}}$.

   Substituting back the optimal solutions (13), it results

   $$\frac{\frac{y+\theta(\eta-\alpha)}{1+\alpha}+\theta}{\frac{\alpha y-\theta(\eta-\alpha)}{1+\alpha}+\eta\theta} = \frac{y+\theta(1+\eta)}{\alpha(y+\theta(1+\eta))} > \frac{1}{\eta^{1/2}} \Rightarrow \eta^{1/2} > \alpha \Rightarrow \eta > \alpha^2$$

3. KUJ: $\xi \equiv u_z/u_e = \frac{e+\eta\theta}{z+\theta}$

   (a) $\frac{\partial\xi}{\partial\theta} = \frac{\eta z - e}{z+\theta}$, which computed at the optimal solutions implies $\frac{\partial\xi}{\partial\theta} > 0$ as long as $\eta > \alpha$, since $z_p^* > e_p^*$.

   (b) $\frac{d\xi}{d\eta} > 0$ and $\frac{d\xi}{d\theta d\eta} > 0$ derive from point (a) with simple algebra.

Lemma 2

1. Consider the unconstrained problem; simple algebra implies

   $$v^p(y_t^i) \equiv \ln\left(\frac{y_t^i + \theta(1+\eta)}{1+\alpha}\right) + \alpha\ln\left(\frac{\alpha(y_t^i + \theta(1+\eta))}{1+\alpha}\right) >$$
   $$\ln\left(\frac{y_t^i}{1+\alpha}\right) + \alpha\ln\left(\frac{\alpha y_t^i}{1+\alpha}\right) \equiv v^{np}(y_t^i) \Rightarrow$$

   $$\Rightarrow \ln\left(y_t^i + \theta(1+\eta)\right) > \ln y_t^i \Rightarrow \theta(1+\eta) > 0 \Rightarrow \forall \theta, \eta > 0, v^p(y_t^i) > v^{np}(y_t^i)$$

2. Using first order conditions (13):

   $$v^p(y_t^i)\mid_{e>0} \equiv \ln\left(\frac{y_t^i + \theta(1+\eta)}{1+\alpha}\right) + \alpha\ln\left(\frac{\alpha(y_t^i + \theta(1+\eta))}{1+\alpha}\right)$$
   $$> \ln\left(\frac{y_t^i + \theta(1+\eta)}{1+\alpha}\right) + \alpha\ln(\eta\theta) \equiv v^p(y_t^i)\mid_{e=0} \Rightarrow$$

   $$\Rightarrow \alpha\ln\left(\frac{\alpha}{1+\alpha}\right) + \alpha\ln(y + \theta(1+\eta)) > \alpha\ln(\eta\theta) \Rightarrow (y+\theta) > \frac{\eta\theta}{\alpha} \Rightarrow y > \frac{\theta(\eta-\alpha)}{\alpha}$$, which is always verified whenever the indirect utility function $v^p(y_t^i)\mid_{e>0}$ is active; that is, whenever $y > \tilde{y} \equiv \frac{\theta(\eta-\alpha)}{\alpha}$.

Proposition 1

Using (10) and (13), it must be verified that

$$v^p\left(y_t^i\right) \equiv u\left(\kappa, 0; \theta\right) = \ln\left(\frac{y_t^i + \theta\left(1 + \eta\right)}{1 + \alpha}\right) + \alpha \ln\left(\eta\theta\right) >$$

$$\ln\left(\frac{y_t^i}{1 + \alpha}\right) + \alpha \ln\left(\frac{\alpha y_t^i}{1 + \alpha}\right) = u\left(z_t^{i,np}, e_t^{i,np}; 0\right) \equiv v^{np}\left(y_t^i\right)$$

After simple algebra, one finds

$$\ln\left(y_t^i + \theta\left(1 + \eta\right)\right) - \ln\left(1 + \alpha\right) + \alpha \ln\left(\eta\theta\right) > \left(1 + \alpha\right)\ln y_t^i + \alpha \ln \alpha - \left(1 + \alpha\right)\ln\left(1 + \alpha\right) \Rightarrow$$

$$V_p\left(y_t^i\right) \equiv \frac{\left[y_t^i + \theta\left(1 + \eta\right)\right]}{\left(1 + \alpha\right)}\left(\eta\theta\right)^\alpha > \left(y_t^i\right)^{1+\alpha}\frac{\alpha^\alpha}{\left(1 + \alpha\right)^{\left(1+\alpha\right)}} \equiv V_{np}\left(y_t^i\right) \qquad\text{(A.1)}$$

Consider the two functions, $V_p\left(y_t^i\right)$ and $V_{np}\left(y_t^i\right)$, which are monotone transformations of the indirect utility functions. The two functions are characterized by the following properties: they are both strictly increasing in y_t^i $\left(V_p'\left(y_t^i\right), V_{np}'\left(y_t^i\right) > 0\right)$; V_p is linear in y_t^i $\left(V_p''\left(y_t^i\right) = 0\right)$, while V_{np} is convex $\left(V_{np}''\left(y_t^i\right) > 0\right)$; finally, $V_p\left(0\right) = \frac{\theta(1+\eta)}{(1+\alpha)}\left(\eta\theta\right)^\alpha > 0$ and $V_{np}\left(0\right) = 0$. Given these properties, there exists a level of income $\check{y}$ such that for each $y_t^i < \check{y} \Rightarrow V_p > V_{np}$, and for each $y_t^i > \check{y} \Rightarrow V_p < V_{np}$. Graphically,

Figure 8: Proposition 1
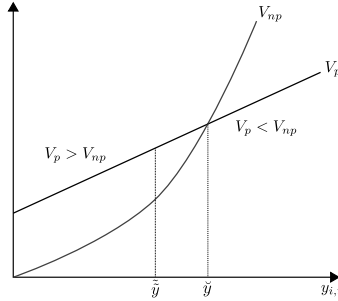
Next, evaluating (A.1) at the level of income $\tilde{\tilde{y}}$, it results that

$$V_p\left(\tilde{\tilde{y}}\right) \equiv \frac{\left(\eta\theta\right)\left(1 + \alpha\right)}{\left(1 + \alpha\right)\alpha}\left(\eta\theta\right)^\alpha > \left(\frac{\theta\left(\eta - \alpha\right)}{\alpha}\right)^{\left(1+\alpha\right)}\frac{\alpha^\alpha}{\left(1 + \alpha\right)^{\left(1+\alpha\right)}} \equiv V_{np}\left(y_t^i\right) \Rightarrow$$

$$\Rightarrow \left(\eta\theta\right)^{1+\alpha} > \left(\frac{\theta\left(\eta - \alpha\right)}{1+\alpha}\right)^{1+\alpha} \Rightarrow \eta\theta\left(1 + \alpha\right) > \theta\left(\eta - \alpha\right) \Rightarrow \alpha\theta\left(1 + \eta\right) > 0, \text{ which is verified for all}$$

$\alpha, \theta, \eta > 0$. This implies that also for $\tilde{y} < \tilde{\tilde{y}}$, $V_p > V_{np}$ and hence $v^p > v^{np}$. Finally, for $\tilde{y} > \tilde{\tilde{y}}$, Lemma 2 ensures that participating will be yet an optimal solution.
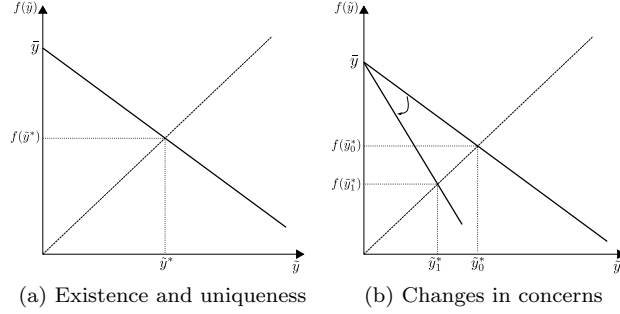
Proposition 2

Existence. Let consider the correspondence $\tilde{y} = f\left(\tilde{y}\right)$ as indicated in (19). If $G_t\left(\tilde{y}_t\right)$ is continuous, the continuity of $f\left(\tilde{y}\right)$ implies the existence of a fixed point of the map from $\tilde{y}$ to $f\left(\tilde{y}\right)$.

Uniqueness. It follows from the monotonicity of $f(\tilde{y})$, which is strictly decreasing ($f'(\tilde{y}) < 0$).
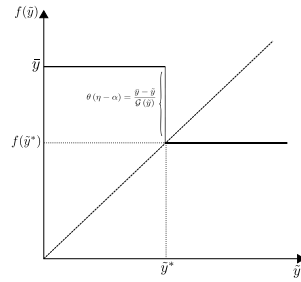
Graphically,

Figure 9: Relative poverty line

(a) Existence and uniqueness   (b) Changes in concerns

Notice, finally, that if $G_t(\tilde{y}_t)$ is a discontinuous function, the existence of $\tilde{y}$ is guaranteed if only if $\exists \theta, \eta, \alpha : \theta(\eta - \alpha) = \dfrac{\bar{y} - \tilde{y}}{G(\tilde{y})}$.
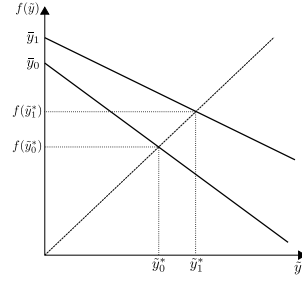
Figure 10: Robustness

Proposition 3

1. Differentiating (19) with respect to the mean income

$$\frac{\partial \tilde{y}_t}{\partial \bar{y}_t} = 1 - \theta(\eta - \alpha)\left[g_t(\tilde{y}_t)\frac{\partial \tilde{y}_t}{\partial \bar{y}_t} + G_{\bar{y}_t}(\tilde{y}_t)\right] \tag{A.2}$$

Simplifying, (20) derives.

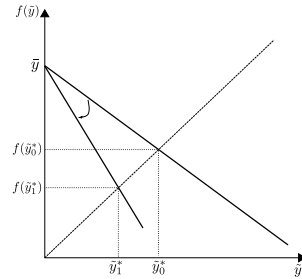Graphically,

Figure 11: Changes in mean income

2. Differentiating (19) with respect to the standard deviation

$$\frac{\partial \tilde{y}_t}{\partial \sigma_t} = -\theta (\eta - \alpha) \left[g_t (\tilde{y}_t) \frac{\partial \tilde{y}_t}{\partial \sigma_t} + G_{\sigma_t} (\tilde{y}_t) \right] \tag{A.3}$$

Simplifying, (21) derives.

Graphically,

Figure 12: Increasing spread

Corollary 1

1. Substituting (20) in (22)

$$\frac{\partial \bar{z}_t}{\partial \bar{y}_t} = \frac{1}{1 + \alpha} - \frac{\theta (\eta - \alpha)}{1 + \alpha} \left[g_t (\tilde{y}_t) \left(\frac{1 - \theta (\eta - \alpha) G_{\bar{y}_t} (\tilde{y}_t)}{1 + \theta (\eta - \alpha) g_t (\tilde{y}_t)} \right) + G_{\bar{y}_t} (\tilde{y}_t) \right]$$

In order to be greater than zero, it must be verified that

$$1 - \theta (\eta - \alpha) G_{\bar{y}_t} (\tilde{y}_t) - \theta (\eta - \alpha) g_t (\tilde{y}_t) \left(\frac{1 - \theta (\eta - \alpha) G_{\bar{y}_t} (\tilde{y}_t)}{1 + \theta (\eta - \alpha) g_t (\tilde{y}_t)} \right) > 0 \Rightarrow$$

$$\Rightarrow 1 - \theta (\eta - \alpha) G_{\bar{y}_t} (\tilde{y}_t) > \theta (\eta - \alpha) g_t (\tilde{y}_t) \left(\frac{1 - \theta (\eta - \alpha) G_{\bar{y}_t} (\tilde{y}_t)}{1 + \theta (\eta - \alpha) g_t (\tilde{y}_t)} \right) \Rightarrow$$

$$\Rightarrow 1 > \frac{\theta (\eta - \alpha) g_t (\tilde{y}_t)}{1 + \theta (\eta - \alpha) g_t (\tilde{y}_t)}.$$

2. Substituting (21) in (23)

$$\frac{\partial \bar{z}_t}{\partial \sigma_t} = -\frac{\theta\left(\eta-\alpha\right)}{1+\alpha}\left[G_{\sigma_t}\left(\tilde{y}_t\right) - g_t\left(\tilde{y}_t\right)\left(\frac{\theta\left(\eta-\alpha\right)G_{\sigma_t}\left(\tilde{y}_t\right)}{1+\theta\left(\eta-\alpha\right)g_t\left(\tilde{y}_t\right)}\right)\right] =$$

$$= -\frac{\theta\left(\eta-\alpha\right)G_{\sigma_t}\left(\tilde{y}_t\right)}{\left(1+\alpha\right)\left(1+\theta\left(\eta-\alpha\right)g_t\left(\tilde{y}_t\right)\right)} < 0$$

Lemma 3

The first three properties are easily verified by simple algebra. The proof of the fourth properties derives by substituting (19) in $\phi^L\left(\tilde{y}_t\right)$; it results that $\phi^M\left(\tilde{y}_t\right) \equiv \left(\eta\theta\right)^\beta > \left(\frac{\alpha}{1+\alpha}\right)^\beta \left(\bar{y}_t - \theta\left(\eta-\alpha\right)G_t\left(\tilde{y}_t\right)\right)^\beta \equiv \phi^L\left(\tilde{y}_t\right) \Rightarrow \bar{y}_t < \frac{\left(1+\alpha\right)\left(\eta\theta\right)}{\alpha} + \theta\left(\eta-\alpha\right)G_t\left(\tilde{y}_t\right) \Rightarrow \bar{y}_t < \frac{\alpha\theta\left(\eta-\alpha\right)G_t\left(\tilde{y}_t\right)+\left(1+\alpha\right)\eta\theta}{\alpha}$, which is always verified whenever the economy is in the first regime, when condition (25) holds, since $\left(1+\alpha\right)\eta\theta > \theta\left(\eta-\alpha\right)$. Further, simple algebra returns $\phi^M\left(\tilde{y}\right) \equiv \left(\eta\theta\right)^\beta = \left(\frac{\alpha}{1+\alpha}\right)^\beta \left(\tilde{y} + \theta\left(1+\eta\right)\right)^\beta \equiv \phi^H\left(\tilde{y}\right)$.

Lemma 4

Let consider the equality: $\phi^M\left(\tilde{y}\right) \equiv \left(\eta\theta\right)^\beta = \frac{\theta\left(\eta-\alpha\right)}{\alpha} \equiv \tilde{y}$. $\phi^M\left(\tilde{y}\right) > \tilde{y}$ implies that $\left(\eta\theta\right)^\beta > \frac{\theta\left(\eta-\alpha\right)}{\alpha}$, which further needs that, given $0 < \beta < 1$, either η or θ or both must be lower than some threshold $\overset{\circ}{\eta}$ or $\overset{\circ}{\theta}$. Let define the function $\Phi(\eta,\theta) \equiv \left(\eta\theta\right)^\beta - \frac{\theta\left(\eta-\alpha\right)}{\alpha}$. The implicit function theorem leads to $\frac{d\theta}{d\eta} = -\frac{\partial\Phi\left(.\right)/\partial\eta}{\partial\Phi\left(.\right)/\partial\theta} = -\frac{\alpha\beta\theta^\beta\eta^{\beta-1}-\theta}{\alpha\beta\theta^{\beta-1}\eta^\beta-\left(\eta-\alpha\right)} < 0$, since $sign[\text{num}] = sign[\text{den}]$.

Lemma 5

$\phi^{H'}\left(y_t^i\right) = \left(\frac{\alpha}{1+\alpha}\right)^\beta \beta \left(y_t^i + \theta\left(1+\eta\right)\right)^{\beta-1}$ evaluated at $\tilde{y}$ gives $\phi^{H'}\left(\tilde{y}\right) = \left(\frac{\alpha}{1+\alpha}\right)\beta\left(\eta\theta\right)^{\beta-1}$. $\phi^{H'}\left(\tilde{y}\right) < 1 \Rightarrow \eta\theta > \left(\frac{\alpha\beta}{1+\alpha}\right)^{\frac{1}{1-\beta}}$. From Proposition 5, the economy will approach the stage II of regime I, in which the followings hold contemporaneously: $\phi^M\left(\tilde{y}_t\right) \equiv \left(\eta\theta\right)^\beta > \bar{y}_t - \theta\left(\eta-\alpha\right)G_t\left(\tilde{y}_t\right) \equiv \tilde{y}_t$ and $\phi^L\left(\tilde{y}_t\right) \equiv \left(\frac{\alpha}{1+\alpha}\right)^\beta \left(\bar{y}_t - \theta\left(\eta-\alpha\right)G_t\left(\tilde{y}_t\right)\right)^\beta < \bar{y}_t - \theta\left(\eta-\alpha\right)G_t\left(\tilde{y}_t\right)$. Rearranging, $\left(\frac{\alpha}{1+\alpha}\right)^{\frac{\beta}{1-\beta}} + \theta\left(\eta-\alpha\right)G_t\left(\tilde{y}_t\right) < \bar{y}_t < \left(\eta\theta\right)^\beta + \theta\left(\eta-\alpha\right)G_t\left(\tilde{y}_t\right)$, which leads to $\left(\eta\theta\right) > \left(\frac{\alpha}{1+\alpha}\right)^{\frac{1}{1-\beta}}$. It derives that it must also hold in the first stage of the economy when $\phi^L\left(\tilde{y}_t\right) > \tilde{y}_t$.