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Heterogeneous entrepreneurs, government quality, and optimal industrial policy

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Abstract

This paper presents a theoretical model exploring the effects of industrial policy (IP) when entrepreneurs are characterized by different ability levels and sectors are heterogeneous as for their profitability and social externalities generated. The optimal structure of IP in terms of monetary transfers is shown to crucially depend on the distribution of entrepreneurs abilities. In an extension of the model, we consider the case in which the Government can use also the provision of business training to entrepreneurs as an additional instrument of IP. Based on these results, policy implication for industrial policy in developing countries are discussed.

Keywords Industrial policy; entrepreneurs; heterogeneous abilities; training.

JEL classification: O25, O15, O14.

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1 Introduction

The recent waves of economic crises have made evident the high vulnerability to external shocks of developed and developing countries. Diversification of production and export has been advocated as a possible strategy to build resilience to shocks. Yet, most of developing countries are highly depended on agriculture and raw material sectors and are experiencing a deindustrialization process (UNCTAD and UNIDO, 2011). As a reaction to this state of affair, there has been an increasing commitment of Governments to support industrialization as part of a broader agenda to diversify the economy through industrial policy (Chang et al., 2013).[1]

Interestingly, this policy change has been accompanied by an increasing agreement among scholars on the fact that - beside the creation of a competitive market environment - Governments of developing countries should also play a proactive role in facilitating structural transformation and industrial upgrading (Yu and Lin, 2015). Economic development requires structural change from low to high productivity activities and the manufacturing sector is a key engine of growth in the development process (Rodrik, 2014). As Ocampo and Ros (2011) point out, once the process of economic growth is seen as a process of structural change, industrial policy becomes a central element of national development strategies.

While industrial policy (IP) is now back in both the political and the economic discourse, this does not imply that it is all clear how to design an effective IP. In fact, there are several factors that may make the effects of IP worse than the problems it aims to solve. In this paper, we emphasize that one of the most relevant obstacle to make IP successful is the mismatch between IP and the quality of the private sector and of the government. In particular, we show how the characteristics of the private sector have important - and not always obvious - implications for the efficacy of IP. The same IP can have even opposite impact depending on the characteristics of the population of entrepreneurs. Learning about the characteristics of the private sector is thus of utmost importance to make IP effective.

To explore these issues, we develop a model whose assumptions have two characteristics: 1) are able to capture those peculiar features of developing economies that are likely to be relevant in determining the impact of IP in those countries and 2) provide a simple setting to study this complex phenomenon. We consider a two-sectors economy populated by agents heterogeneous in terms of entrepreneurial

[1] This phenomenon has been particularity relevant in African countries. The New Partnership for Africa’s Development (NEPAD) adopted by African leaders in 2001 identifies economic transformation through industrialization as a critical vehicle for growth and poverty reduction in the region. In 2008, African heads of state signed the Plan of Action for the Accelerated Industrial Development of Africa (AIDA). In 2010, the Economic Community of West African States (ECOWAS) adopted the West African Common Industrial Policy. Industrialization is a component also of recent national development programmes for a number of countries including Brazil, Egypt, Ethiopia, Kenya, Namibia, Myanmar, Oman, Papua New Guinea, Tunisia, Uganda, and Vietnam (Altenburg and Lutkenhorst, 2015).

abilities. The two sectors differ for the amount of knowledge diffused among the entrepreneurs' population about their specific production process. The already active or *known sector* (e.g. agriculture, tourism etc.) is characterized by a high degree of diffusion among entrepreneurs of the knowledge needed to operate its production process. On the contrary, for the non-active or *unknown sector* (e.g. manufacturing) the knowledge about the production tasks is not yet standardized and/or diffused in the population. This cross-sectoral heterogeneity is reinforced in terms of sector profitability and externality: while the already active or *known sector* provides a higher return for private entrepreneurs than the other, the Government objective is to support the *unknown sector* because it is assumed to produce a higher positive externality.

In this setting, IP is conceived as the set of selective government measures to influence the structure of the economy in order to increase the share of the high social return unknown sector (manufacturing) to maximize the aggregate welfare of the population. We assume that IP is under the responsibility of the Industrial Policy Agency (IPA), the country-specific governmental body which is in charge of the design, implementation and monitoring of the national and subnational industrial policy. We model IP in two ways. The first one is a monetary transfer to entrepreneurs. This is sector specific and independent from the individual productivity of the entrepreneur. The second is instead the IPA increasing the economy-wide knowledge level about the production in the unknown sector. This is achieved by providing local entrepreneurs entering the unknown sector with training, market information, and logistic support to begin the activity and reduce the costs associated with production and exporting in that sector.

This simple model structure allows us to derive a number of results. First, we show that - as long as in the economy there is a positive level of knowledge concerning the production technique of each sector, IP always increases welfare with respect to a neutral policy, i.e. the one that provides the same incentives to both sectors. Second, our results show that the structure of the sector incentives of the optimal IP largely depends on the distribution of entrepreneurs abilities. In particular, the welfare gain effect of IP is larger in economies where the level of knowledge about the production process in the unknown sector is low (relatively to the average ability of the population) and the inequality in the abilities' distribution is high or, on the contrary, where the level of knowledge about the unknown sector is high and inequality is low. Third, when we allow for Government failures, we find that, while the larger the bias of the IPA the lower the welfare gain of the IP with respect to the neutral policy, there always exists a non-empty set of values for the bias for which IP remains beneficial. It follows that there are configurations of parameters for which IP increases welfare even in the presence of a corrupt and non-benevolent Government. Fourth, when we expand the model to allow for IPA to use two instruments, namely the monetary transfer and the support to entrepreneurs entering the unknown sector, we find that the latter and the quality of entrepreneurs are in fact substitutes: the higher is the ability of the private sector, the less needed are investments to improve the capabilities of the IPA in coordinating or guiding the economic activity. At the same time, the higher the inequality in the abilities the more important is the role of the IPA.

Taken together these results support the view that the effectiveness of IP not only depends on the characteristics of the specific IP measures adopted but also on the quality of the entrepreneurs and on the capabilities of the government. It is thus essential to learn about and take into proper account the heterogeneity in the entrepreneurs abilities to identify the IP that would work better in each context.

Three main assumptions simplify our analysis. First, we abstract from the possibility of international trade. It is well known that in the context of an open economy the arguments in favour of IP are weaker. We will show that even in the more favorable closed-economy case the conditions for the optimality of IP may be very stringent. Second, we assume that the Government is benevolent and there is no corruption. Under these assumptions, we are able to show that even excluding any political economy consideration IP optimality is a nonobvious outcome that indeed greatly depends on the fit between IP and the characteristics of the entrepreneurs. Finally, we abstract from local taxation issues since we assume that IP is financed by an external donor. While this is indeed a not too heroic assumption given that most of developing countries heavily rely on donors to finance government programs to support domestic firms (also because of the well-know difficulties to collect taxes), it also allows us to focus only on which is the optimal structure of IP.

Our paper is related to different strands in the literature. The first is the large and heterogeneous literature that argues that manufacturing and structural change matter for economic growth and that sectors are different in terms of productivity and externalities they generate (Rodrik, 2014; Lin, 2013; Szirmai, 2012). This heterogeneous body of research is relevant for our analysis to the extent to which the unknown sector can be considered to be the manufacturing sector. We argue that this is very likely in the context of developing countries where usually the largest share of economic activity takes place in agricultural, service or natural resource sectors.

Our paper is also related to the literature on the role of government intervention and in particular on the effects of industrial policy in developing countries. The empirical literature on the effect of industrial policy has mostly focused on the analysis of the Asian Tigers and the Latin American experiences (see Amsden, 1998; Chang, 1994; Lall, 1996; Noland and Pack, 2002; Di Maio, 2009) while the number of studies on the Sub-Saharan countries experiences with IP are much smaller (for a review see UNECA, 2011). Theoretical research has analysed the effect of IP on growth using a large variety of different models and reaching almost any possible conclusion (Hausmann and Rodrik, 2003; Hodler, 2009; Hoff, 1994; Harrison and Rodrigues-Clare, 2010). In the literature, IP has been modeled in different ways: targeted subsidies; monetary transfers to cover the fixed costs of production; a regulatory policy forcing firms to remain in one specific sector; a subsidy provided only to innovative firms (Bjorvatn and Coniglio, 2006; 2012; Aghion et al., 2015; Hausman and Rodrik, 2003). With respect to this literature, our model consider two types of IP (monetary transfer and improvement in IPA quality) and it is the first one to explore the role of entrepreneur's heterogeneous abilities in determining the optimal IP.[2]

Finally, our paper is close to the set of contributions looking at which policies would favour most entrepreneurship and economic activity. In general, all policies directed to influence entrepreneurs' decisions can be considered as part of IP. There are few empirical papers looking at the micro-economic effects of those policies in developing countries. While economic theory has long focused on mechanisms via which expanded access to capital enables individuals to alter their occupational choices and improve their economic conditions (Banerjee and Newman, 1993), there is an increasing attention to the need to complement this with skill and information provision. Bandiera et al. (2017) emphasize the importance of the process of occupational change and of programs which enable poor people to upgrade occupations, rather than just make them more productive in a given occupation. Lin (2012) argues that governments need to play a proactive role to facilitate structural transformation and industrial upgrading by providing information to entrepreneurs in the private sector on the the most dynamic industries. While there is an increasing evidence on the disappointing performance of short-term training for existing micro-entrepreneurs (McKenzie and Woodruff, 2014), recent evaluations of business training programs provide evidence of the existence of a complementarity between the provision of capital and training (de Mel et al. 2012). Given the importance of both these aspects, our IP actually uses both these instruments: it provides economic incentives in the form of an individual cash transfer and provides support services for entrepreneurs investing in a new sector.
The paper proceeds as follows. In section 2 we present the basics of the model. In section 3 we discuss the optimal IP when the government can use only one measures (individual cash transfer). In Section 4, we extend the basic model allowing for possible Government failures as well as considering the case in which the government can use an additional measure (business training) as part of IP. Section 5 concludes.

[2] Acs and Naude (2013) note that one of the reasons for IP failure is the "inappropriate emphasis on stimulating economic activities and growth in a manner that was not optimal for entrepreneurship given these countries' levels of development". In fact, there is considerable evidence that in countries where IP has been more successful such as the NIEs and China - IP was designed taking into proper account the characteristics of the country's entrepreneurs and their relation to the State. For instance, where the entrepreneurial base was small and weak (Singapore and Korea), IP was at first aimed to complement and strengthen the domestic entrepreneurial base, through allowing in much more foreign entrepreneurship and by providing financial support to allow entrepreneurs to take on more risk in imitation and foreign technology adoption. On the contrary, where the entrepreneurial base was fairly strong to begin with (e.g. Taiwan and Japan), more limitations were placed on foreign entrepreneurs (Nelson and Pack, 1999).

2 The model

2.1 Basic structure

Consider a two-sectors economy populated by a continuum of individuals of mass M with different innate abilities $a_i \in [0,1]$. Individuals (henceforth, the entrepreneurs) are risk-neutral and self-employed in their small or medium size enterprises (SME's). The two sectors differ as for the amount of knowledge diffused among the population about the sector-specific production processes. The already active or known sector (denoted by "*k*") is characterized by a high degree of diffusion among the population of entrepreneurs of the knowledge needed to operate its production process. On the contrary, in the case of the non-active or unknown sector (denoted by "*u*") production knowledge is not yet diffused and/or production tasks are not yet standardized.[3]

Return y_{ij} for project i in sector j is determined by sector-specific productivity π_j and by entrepreneurial skills e_{ij} of individual i operating in sector $j = k,u$, namely: $y_{ij} = \pi_j e_{ij}$. The level of entrepreneurial skills e_{ij} is a weighed average of the ability of the entrepreneur, a_i, and of the level of basic knowledge available in the economy concerning the sector production process, $b_j > 0$. The level of basic knowledge b_j may be determined by several factors, including government intervention, historical episodes, geographical conditions or even chance. Both the ability of the entrepreneur and of the level of basic knowledge concerning the sector production technology positively contribute to the project return. However, their weight differ in each sector. While in the known sector, project return hinges relatively more on the entrepreneur's individual ability (since the level of knowledge about the sector is already high and diffused), in the unknown sector the project return depend more on the entrepreneur having the basic knowledge about the production process (since entrepreneur's ability is not very useful if the basics are not known). Formally, $y_{ij} = \pi_j e_{ij}$, where $e_{ij} = \theta_j a_i + (1 - \theta_j) b_j$ with $j = u,k$ and $\theta_k > \theta_u$. To simplify the exposition, we assume $\theta_k = 1$ and $\theta_u = 0$, i.e. that in the known sector return only depends on the entrepreneurs' ability, a_i, while in the unknown what matters is only the amount of basic knowledge available, b, in the economy which the entrepreneur can access fully and at no cost.[4]

[3] This formalization is meant to capture the situation - common to several developing countries - in which entrepreneurs have often deep knowledge about the traditional sector (agriculture or often also in the tourism and natural resources sectors) while there is a significant lack of expertise in the non-traditional sectors (in most of the cases manufacturing).
[4] In fact, the only necessary condition for all our results to hold true is that the basic knowledge component is relatively more important in the unknown sector rather than in the known one.

2.2 Industrial Policy

As we discussed in the Introduction, Industrial policy (IP) can be defined as the set of selective government measures to modify the structure of the economy by increasing the share of the manufacturing sector. IP can take different forms including subsidies, tax concessions, soft loans, preferential procurement policies, import restrictions or export promotion measures. We assume that IP is under the responsibility of the Industrial Policy Agency (IPA), the country-specific governmental body in charge of the design, implementation and monitoring of the national and sub-national industrial policy (Hodler, 2011).[5]

In our model, IP takes two forms. The first one is a monetary transfer to entrepreneurs.

This is sector specific and independent on the individual productivity of the entrepreneur. The second is the IPA increasing the economy-wide knowledge level about the production in the unknown sector. This is achieved by providing local entrepreneurs entering the unknown sector with training, market information, and logistic support to begin the activity and reduce the costs associated with production and exporting in that sector.[6] In this case, the level of basic knowledge concerning the production process in the unknown sector, i.e. b, is a choice variables of the IPA maximization problem. To simplify the exposition of the model, we begin by assuming that b is exogenously given. In Section 4.2, we consider the case in which IPA uses both instruments and thus we endogenize b in the budget constraint.

We assume that IPA has an exogenously given amount G of resources (e.g. provided by an international donor development project) to be used to conduct IP (Hodler, 2011). The total amount of resources G is fully utilized, so there is no saving by the IPA. The individual transfer received by each entrepreneur in the known sector is τ_k and the total amount at the sector level is:

$$T_k := M\eta\tau_k$$

where η is the share of entrepreneurs investing in the known sector. The amount of resources received by each entrepreneur in the unknown sector is τ_u and the total amount is:

$$T_u := M(1-\eta)\tau_u$$

[5] For a historical overview and empirical assessment of IPA activities and strategies in different countries see for instance Amsder, 2001; Cimoli et al. 2009; Naude et al. 2015.

[6] For instance, these are the activities usually performed by export promotion agencies in several developing countries (Belloc and Di Maio, 2012).

where $(1-\eta)$ is the share of entrepreneurs investing in the unknown sector.
When b is exogenous, IP is characterized by the couple $(\tau_k;\tau_u)$. Hence the budget constraint of the IPA is simply given by:

$$G = M(\eta\tau_k + (1-\eta)\tau_u) \tag{1}$$

that rewritten in per-capita terms is:

$$g = \eta\tau_k + (1-\eta)\tau_u \tag{2}$$

with $g := G/M$ defined as the per-capita resources available for IP.

2.3 Entrepreneur sectoral allocation choice

As it follows from our assumptions (see Section 2.1), the entrepreneurial skill is $e_{ij} = a_i$ if $j = k$, and $e_{ij} = b$ if $j = u$. Hence, the profit associated with a project in the known sector for entrepreneur i is:

$$\pi_k\, a_i + \tau_k$$

while the profit associated with a project in an unknown sector is:

$$\pi_u b + \tau_u$$

The entrepreneur chooses to invest in the known sector if and only if

$$a_i \geq \bar{a} := \frac{\pi_u b + \Delta\tau}{\pi_k} \tag{3}$$

with $\Delta\tau \equiv \tau_u - \tau_k$. This condition gives the threshold ability level $\bar{a}$ that identifies entrepreneurs investing in the known sector: all those with an individual ability higher than $\bar{a}$ will have a higher (after the transfer) profit if they invest in the known sector. The reverse holds for low ability individuals.
For any a_i, we denote $F(a_i)$ as the share of entrepreneurs that have individual ability smaller or equal than a_i. Formally, if f denotes the density of the share of the entrepreneurs ability distribution, for any entrepreneurial skill $\tilde{a}$, we have:

$$F(\tilde{a}) = \int_0^{\tilde{a}} f(a_i)\,\mathrm{d}a_i \tag{4}$$

Using this notation, the share of entrepreneurs in the known and unknown sectors can be, respectively, written as $\eta = \int_{\bar{a}}^1 f(a_i)da_i$ and $(1-\eta) = \int_0^{\bar{a}} f(a_i)da_i$. It follows that the sector-level knowledge structure (in particular the level of knowledge in the unknown sector b) plays a central role in determining the economy patter of specialization.

2.4 *Aggregate welfare*

Aggregate welfare is given by the sum of total sectoral returns, i.e. the social return of each investment plus the transfer. We assume that investment in any of the two sectors generates both a private return and a positive externality. Thus, the social return of each investment Π_j, with $j = k,u$, is given by

$$\Pi_u = \pi_u + U > \pi_u \qquad \text{and} \qquad \Pi_k = \pi_k + K > \pi_k \tag{5}$$

where $U,K > 0$ are the sector-specific externalities of the unknown and known sector, respectively.[7] Each investment in the known sector generates a total sectoral return equal to

$$\Pi_k a_i + \tau_k$$

i.e. the sum of the ability-weighted social return of the investment (individual return plus the externality) and of the individual transfer received by the entrepreneur. It follows that the total welfare generated by projects in the known sector is given by

$$M\left(\Pi_k \int_{\bar{a}}^{1} a_i f(a_i)\,\mathrm{d}a_i + \int_{\bar{a}}^{1} \tau_k f(a_i)\,\mathrm{d}a_i\right)$$

The total return for each unknown project is, instead, given by

$$\Pi_u b + \tau_u$$

so that the total welfare generated by projects in the unknown sector is

$$M(\Pi_u b + \tau_u)\int_0^{\bar{a}} f(a_i) da_i.$$

Finally, the net per capita aggregate welfare, defined as the average of all individual returns and transfers, is

[7] Note that this simple formalization allows for the consideration of several different cases, including that in which a sector with low individual productivity π_j can generate high individual total returns because of its high positive externality.

$$\frac{W}{M} \equiv \omega = \Pi_k \int_{\bar{a}}^{1} a_i f(a_i)\, \mathrm{d}a_i + \int_{\bar{a}}^{1} \tau_k f(a_i)\, \mathrm{d}a_i + (\Pi_u b + \tau_u) \int_0^{\bar{a}} f(a_i) da_i$$
$$= \Pi_k \left(\int_{\bar{a}}^{1} a_i f(a_i)\, \mathrm{d}a_i \right) + b\Pi_u \int_0^{\bar{a}} f(a_i) da_i + g \quad (6)$$

Equation (6) makes clear that, in our model, IP influences aggregate welfare in two ways.
First, IP directly increases welfare because each individual receives a transfer g at no cost. Second, IP indirectly affects welfare by modifying the allocation of entrepreneurs among the two sectors. Note that, differently from other models, in our case IP does not affect welfare by modifying sector productivities. This choice allows us to explore in detail the role of IP as an instrument for creating the conditions for structural change through the reallocation of individuals across different economic activities.

3 Results

We begin our analysis by deriving the optimal IP when the IPA is benevolent and has perfect information, i.e. there are no government failures. This implies that IPA knows the *true* values of the sector-specific externalities.[8] The analysis of the effect of government failures on the characteristics of the optimal IP is presented in Section 4.1.

3.1 Optimal Industrial Policy

The objective of IPA is to select the sectoral allocation of total resources G that maximizes aggregate welfare. Thus, the IPA chooses τ_k and τ_u that maximize (6), by inducing the optimal sectoral allocation of entrepreneurs which satisfies (3). Since the choice of τ_k and τ_u impacts the welfare only through the value of $\bar{a}$, i.e. the threshold ability level that determines the sector allocation of entrepreneurs, we first find the optimal level of $\bar{a}$ by maximizing eq. (6). This is given by

$$\bar{a}^* = \frac{\Pi_u b}{\Pi_k} \quad (7)$$

This is the ability value for which the return of the marginal entrepreneurs in the know and unknown sector is equalized while individuals with ability greater (resp. smaller) than $\bar{a}^*$ invest in the known (resp. unknown) sector.

[8] The values of U and K employed by IPA in choosing IP can be the result of different processes, ranging from being the outcome of a correct economic analysis, of a politically motivated decision (the values are determined by the objective function of the Government) or of the activity of lobby groups on the Government. In fact, as with any other form of government intervention, the effectiveness of IP can be seriously impaired by the presence of different sources of government failure.

Using (3) and (5), the optimal allocation condition (7) can be rewritten as:

$$b\left(\frac{U\pi_k - K\pi_u}{\pi_k + K}\right) = \tau_u - \tau_k \tag{8}$$

The optimal IP is characterized by non-negative individual transfers τ_u and τ_k that satisfy the budget constraint in eq. (1). There are three possible cases: 1) corner solution with $\tau_u = 0$; 2) corner solution with $\tau_k = 0$; 3) internal solutions with both transfers positive. The first two cases correspond to situations where the condition (8) cannot be satisfied because - for given parameters - g is not large enough and, hence, the optimal allocation of entrepreneurs described in (7) is not feasible.[9]
Focusing on the internal solution case, we derive the following proposition:[10]

Proposition 3.1. *The optimal τ_k and τ_u are the solutions of*

$$\begin{cases} b\left(\frac{U\pi_k - K\pi_u}{\pi_k + K}\right) = \tau_u - \tau_k \\ g = \eta\tau_k + (1-\eta)\tau_u = \left[1 - F\left(\frac{\Pi_u b}{\Pi_k}\right)\right]\tau_k + F\left(\frac{\Pi_u b}{\Pi_k}\right)\tau_u \end{cases} \tag{9}$$

and (7) is satisfied so that

$$\tau_u = g + [1 - F(\bar{a}^*)]\, b\left(\frac{U\pi_k - K\pi_u}{\pi_k + K}\right) \tag{10}$$

$$\tau_k = g - F(\bar{a}^*)\, b\left(\frac{U\pi_k - K\pi_u}{\pi_k + K}\right) \tag{11}$$

Proposition 3.1 shows that the optimal IP crucially depends on the value of the sector externality as assumed by the Government. To focus on the most interesting case, we assume that the positive externality generated by an investment in the unknown sector is larger than that in the known sector, i.e. $U > K$.[11]To make the analysis clearer, we assume $U > 0$ and $K = 0$. Under these assumptions, the optimal IP is characterized by the individual transfers

[9] It is possible to show that the IP induces the optimal allocation of entrepreneurs only under the condition that a minimum level of transfers is available. Note that the larger is the value of g, the larger is the possibility for the IPA to induce a reallocation of entrepreneurs. Yet, for g larger than a certain critical value $\bar{g}$, entrepreneurs' choices are unaffected, so that the marginal entrepreneur has ability equal to $\bar{a}^*$ in eq. (7). It follows that the value of $\bar{g}$ divides the corner solutions cases and the internal solutions ones.
[10] The full characterization of the optimal IP for the different cases is derived in Appendix A, see in particular Proposition A.1.
[11] Note that this is the configuration for which a-priori IP may appear less controversial. By showing that also in this case optimal IP largely depends on the distribution of entrepreneurs abilities strengthen our argument.

$$\tau_u = \quad g + [1 - F(\bar{a}^*)]bU, \tag{12}$$

$$\tau_k = \qquad g - F(\bar{a}^*)bU, \tag{13}$$

while the optimal amounts of resources transferred to each sector are

$$T_u = M(1 - \eta)\tau_u = MF(\bar{a}^*)(g + (1 - F(\bar{a}^*))bU) \tag{14}$$

$$T_k = M\eta\tau_k = M(1 - F(\bar{a}^*)(g - F(\bar{a}^*)bU) \tag{15}$$

It can be easily verified that, for $K = 0$, the minimum level of per-capita resources $\bar{g}$, which guarantees that the optimal allocation of entrepreneurs $\bar{a}^*$ is feasible, is given by:

$$\bar{g} := F\left(\frac{\Pi_u b}{\Pi_k}\right) bU. \tag{16}$$

Aggregate Welfare The IP affects aggregate welfare through two channels. On one side, IP increases aggregate welfare by providing monetary transfers to all entrepreneurs. On the other side, IP impacts on aggregate welfare by providing (asymmetric) incentives at the sector level to influence the size and the ability composition of the group of entrepreneurs operating in the two sectors.

To disentangle the two phenomena, we define as benchmark the aggregate welfare when the IPA chooses the neutral policy $\tau_u = \tau_k = g$.[12] It is easy to show that:

Proposition 3.2. *For any $b > 0$, the aggregate welfare under optimal IP is always strictly larger than under the neutral policy.*

As long as the level of basic knowledge in the unknown sector is not zero, the neutral policy (same transfer to both sector) does never satisfies the optimality conditions (12) and (13), and hence it is always suboptimal. This result implies that the optimal IP does not necessarily need to target only sectors characterized by dynamic comparative advantage *á la* Redding (1999). In our setting, indeed,

[12] Observe that such a policy is admissible (the contribution to each entrepreneur is g and the total cost of the intervention is $G = Mg$) and non-distortionary (entrepreneur i chooses to invest in a project in the known sector if and only if $a_i \geq (\pi_u b + \Delta\tau)/\pi_k$ where $\Delta\tau \equiv \tau_u - \tau_k = 0$, which is equivalent to $a_i\pi_k \geq \pi_u b$, i.e. the condition to invest in a project in the known sector in absence of IPA intervention).

the welfare increasing effect of IP follows specifically from the fact that IP induces low-ability entrepreneurs to invest in the unknown sector.[13]

3.2 *Optimal IP and distribution of entrepreneurs' abilities*

In order to characterize in detail the link between the optimal IP and the characteristics of the economy, in the following we explore how the former changes with the distribution of entrepreneurs' abilities. More precisely in this section, we analyze how optimal IP changes with differences in the: 1) average population ability and; 2) inequality in the ability distribution (for given average ability level of the population of entrepreneurs).

Comparing populations of entrepreneurs with different average ability levels We consider a family of cumulative distributions of entrepreneurs' abilities F_γ indexed by a real parameter γ. We assume that the distributions are "ordered" in the sense that $F_\gamma(\bar{a}^*)$ is decreasing in γ. This is the case, for instance, if F_γ are ordered in the sense of stochastic dominance. We denote with η_γ the share of entrepreneurs investing in the known sector when the distribution of ability is induced by F_γ. We have the following results:

Lemma 3.3. *Under the optimal IP, η_γ is increasing in γ.*

Proof. See Appendix A. □

Proposition 3.4. *The optimal individual and sectoral transfers are characterised as follows:*

(i) If $g \geq F_\gamma\left(\frac{\Pi_u b}{\Pi_k}\right) bU$, individual transfers τ_k and τ_u are increasing in γ; also, T_k (T_u) is increasing (decreasing) in γ.

(ii) If $g \leq F_\gamma\left(\frac{\Pi_u b}{\Pi_k}\right) bU$, $\tau_u = \frac{g}{F\left(\frac{\Pi_u b}{\Pi_k}\right)}$ and $T_u = g$, while $\tau_k = T_k = 0$.

Proof. See Appendix A. □

Proposition 3.4 illustrates how - when IPA has enough resources (case *i*) - the optimal IP changes with the average level of ability in the entrepreneurs' population. If the ability level of the population is low, it is optimal for the IPA to support relatively more the entrepreneurs in the unknown sector (the lower γ the higher T_u). On the contrary, for high levels of average ability, the optimal IP is

[13] Those are all individuals with entrepreneurial ability smaller than the level of the basic knowledge in the unknown sector ($a_i < b$), implying that for them it always holds $y_u > y_i$.

characterized by more resources transferred to the known sector. It follows that - *ceteris paribus* - the optimal structure of the IP incentives depends on the average ability level of the entrepreneurs' population.

While IP is always welfare improving as long as $b > 0$ (see Proposition 3.2), it is important to evaluate how this effect changes with the ability level in the population. To analyze how the welfare gain of the optimal IP varies with respect to the benchmark case (i.e. neutral policy) as a function of γ, we define the gain in the aggregate per-capita welfare generated by the optimal IP, as following:

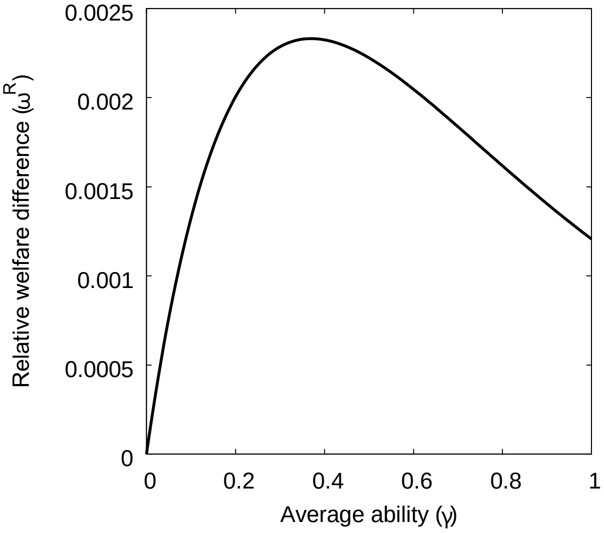
$$\omega^R = \frac{\omega^*}{\omega^n} - 1 \qquad (17)$$

where, using eq. (6), $\omega^* = \omega(\bar{a}^*)$ is the welfare under the optimal IP (i.e., evaluated at $\bar{a}^*$), while $\omega^n = \omega(\bar{a}^n)$ is the welfare under the neutral policy, (i.e., evaluated at $\bar{a}^n = \pi_u b/\pi_k$).

To numerically solve the model, we consider a family of cumulative abilities $F_\gamma(a) = a^\gamma$ for $\gamma \in [0,1]$, with larger γs associated with higher abilities (in the sense of first order dominance).[14] Figure 1 shows the relation between ω^R and γ for different distributions of abilities. For small γ's, the welfare effect of the optimal IP is always increasing in the population abilities. To understand why, we first note from eq. (6) that the IP has two opposite effects on welfare. On a side, the higher the number of entrepreneurs in the unknown sector (i.e., the lower is γ), the larger the positive welfare effect of supporting them (*size effect*). On the other side, the optimal IP - providing a higher transfer to entrepreneurs in the unknown sector - has a negative effect on welfare since it induces some high ability individuals to divert their investment from the known (high productivity) sector to the unknown (low productivity) one (*productivity effect*). Starting from small γ, the size effect is greater than the productivity one. As γ increases, there exists a threshold ability level beyond which the productivity and the number of entrepreneurs in the known sector become large enough that supporting relatively more the entrepreneurs in the unknown sector does not relatively increase welfare: after this point, the welfare gain of IP is smaller the larger the γ.

[14] This functional form generates in a simple way a positive correlation between average ability of the population and individual returns (see Section 2.1).

Figure 1: Relative welfare difference (ω^R): optimal IP vs benchmark



Parameters: $\pi_k = \Pi_k = 1.5$, $\pi_u = 1$, $U = 1$, $b = 0.1$, $g = 2$.

Summarizing, the welfare gain effect of IP is larger for economies with entrepreneurs with intermediate level abilities: the allocation induced by the optimal IP and the benchmark one are the most different. On the contrary, for very small or large values of γ, the optimal and benchmark allocation are more similar and the welfare gain of IP is small.

Comparing population of entrepreneurs characterized by different inequality levels in the ability distribution We now compare the optimal IP for populations of entrepreneurs characterized by different inequality levels in the abilities' distribution, while holding constant the average ability. Due to the analytical complexity of this comparison, we consider the following specific family of cumulative distribution function:

$$F_\alpha(a) = a + \alpha\varphi(a) \tag{18}$$

where $\varphi(a)$ is the function

$$\varphi(a) = a(a - 1/2)(a - 1). \tag{19}$$

While for any value of α the average value of the ability in the population is always 1/2, the larger is α the higher is the inequality in the abilities' distribution.[15] Using eqs. (18) and (19), we start noticing that:

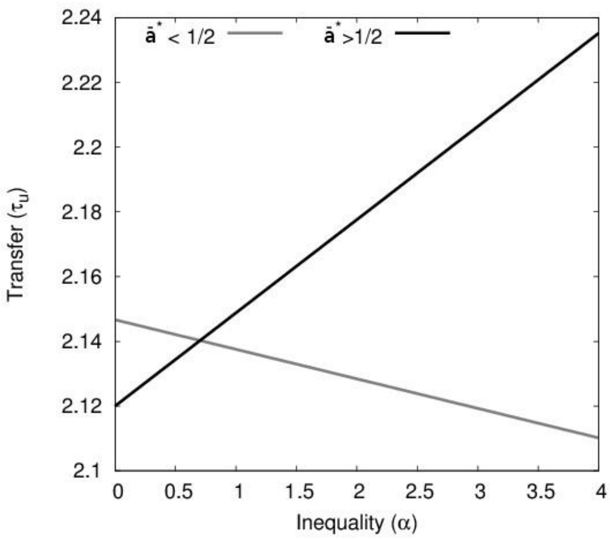
[15] This functional form of the cumulative distribution function (18) connects, in a simple way, the mean preserving spread of the distribution to one parameter only (i.e., α). The distribution is ordered in the sense of second order

$$\frac{\partial F_\alpha(\bar{a}^*)}{\partial \alpha} \ if \ and \ only \ if \ \ \bar{a}^* \lesseqgtr \frac{1}{2} \qquad (20)$$

Condition (20) indicates that for an economy in which the ability of the marginal entrepreneur ($\bar{a}^*$, see eq. 7) is lower (resp., higher) than the average ability level of population, a higher inequality is associated with a larger (resp., smaller) number of entrepreneurs investing in the unknown sector.[16]

The implications of this mechanism for the structure of optimal IP for different levels of inequality in the distribution of entrepreneurs' abilities are illustrated in Figure 2. Numerical results show that the relation between the optimal individual transfer and the distribution of entrepreneurial ability depends on the knowledge structure of the economy. In economies where the level of basic knowledge *b* is low enough such that the ability of the marginal entrepreneur is lower than the average ability in the population (i.e. $\bar{a}^* < 1/2$), the higher is the inequality the smaller is the optimal individual transfer to entrepreneurs in the unknown sector (τ_u). On the contrary, if the economy has a level of *b* high enough such that $\bar{a}^* > 1/2$, the higher the inequality the higher the optimal transfer τ_u.

Figure 2: Optimal individual transfer to the unknown sector (τ_u) and inequality (α): low and high ability of the marginal entrepreneur ($\bar{a}^*$)



Parameters: $\pi_k = \Pi_k = 1.5$, $\pi_u = 1$, $U = 1$, $g = 2$; $\bar{a}^* < 1/2$ ($b = 0.2$), $\bar{a}^* > 1/2$ ($b = 0.6$).

As it directly follows from eqs. (12)-(13) and (20), if $\bar{a}^* < 1/2$, a higher inequality implies a higher share of entrepreneurs working in the unknown sector, and thus a lower amount of resources that can

stochastic dominance (e.g., Lorenz dominance). The choice of the mean at 1/2 is a simplifying assumption which does not qualitatively affect the results.

[16] This directly follows from the fact that an increase in the spread makes the tails of abilities distribution fatter.

be transferred to each of them.[17] At the same time, a higher inequality corresponds to a higher minimal level of resources necessary to induce the optimal allocation of entrepreneurs (see 16). The vice-versa holds for $\bar{a}^* > 1/2$. Hence, depending on the level of basic knowledge in the unknown sector (which is a crucial determinant of $\bar{a}^*$), inequality in the abilities' distribution may generate either an additional cost or a bonus for the optimal IP, with higher inequality resulting more costly for economies with low levels of basic knowledge.

This mechanism becomes even clearer if we analyze in another way the role of inequality by comparing two economies with same abilities' distributions but different levels of basic knowledge in the unknown sector. As it follows from eqs. (12)-(13), a higher level of b has two opposite effects. On the one hand, through its effect on the ability of marginal entrepreneur ($\bar{a}^*$), a higher b is indirectly associated with a higher number of entrepreneurs working in the unknown sector and thus with a lower per-capita transfer τ_u (the same *size effect* described above). On the other hand, a higher b also increases the productivity of each entrepreneur in the unknown sector (*productivity effect*), pushing the differential $\tau_u - \tau_k$ (eq. 9), by directly increasing (resp., reducing) the level of τ_u (resp., τ_k). Greater inequality tends to amplify the size effect, and then the cost associated with a larger number of entrepreneurs in the unknown sector. For sufficiently low levels of inequality the size effect is lower than the productivity one, so that economies with low levels of basic knowledge in the unknown sector can support higher optimal individual transfers than economies with high levels of basic knowledge can do. On the contrary, for sufficiently high levels of inequality, the relation is reversed: economies with low levels of basic knowledge in the unknown sector show smaller individual transfers than economies with high levels of basic knowledge.

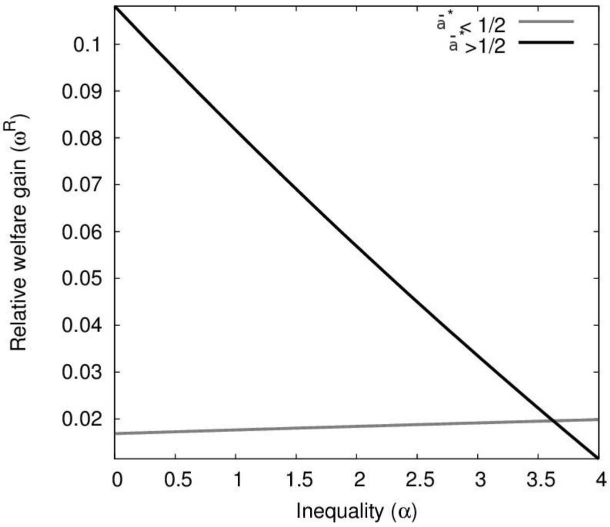
Together, these opposite results demonstrate once again how sensitive the optimal IP is to the actual distribution of abilities in the population of entrepreneurs and, as a consequence, how diverse can be the impact of a given IP on aggregate welfare depending on the inequality in the ability levels and on the knowledge structure of the economy. Specifically, Figure 3 illustrates that the welfare gain (ω^R) induced by the optimal IP (relatively to the neutral policy) is either larger or smaller in the level of inequality α depending on whether $\bar{a}^* < 1/2$ or $\bar{a}^* > 1/2$.

When the level of basic knowledge is high, the ability of the marginal entrepreneur is higher than the average ability of the population ($\bar{a}^* > 1/2$). In this case, a large share of entrepreneurs are active in the unknown sector, while only the highest ability individuals invest in the known sector. In this environment, the IP, which is characterized by an asymmetric treatment of entrepreneurs ($\tau_u > \tau_k$), has a larger effects on aggregate welfare than the neutral policy ($\tau_u = \tau_k$). However, the higher the

[17] Recall that the unknown sector is always more costly for IPA than the known sector and that the difference $\tau_u - \tau_k$ does not depend on α (see eq. 9).

inequality, the smaller the number of entrepreneurs in the unknown sector (eq. 20), and then the smaller the effect of IP on aggregate welfare. In this case, the welfare gain of the optimal IP decreases with inequality. When the level of basic knowledge is low (i.e., $\bar{a}^* < 1/2$), the effects are reversed.

Figure 3: Relative welfare difference and inequality: low and high ability of the marginal entrepreneur



Parameters: $\pi_k = \Pi_k = 1.5$, $\pi_u = 1$, $U = 1$, $g = 2$; $\bar{a}^* < 1/2$ ($b = 0.2$), $\bar{a}^* > 1/2$ ($b = 0.6$).

A large part of the entrepreneurs invest in the known sector, while only few choose the unknown one: this implies that the IP generates only small gains on aggregate welfare with respect to a neutral policy. Yet, the higher the inequality, the higher the fraction of entrepreneurs in the unknown sector (eq. 20), and then the larger becomes the role of the optimal IP in sustaining aggregate welfare. Hence, in this case, the welfare gains of the optimal IP increases with inequality.

This mechanism suggests that IP plays a more important role in countries where either the level of basic knowledge concerning the unknown sector is high and inequality in the entrepreneur's ability distribution is low, or, on the contrary, where the level of basic knowledge in low and inequality is high. To understand why, note that IP impacts aggregate welfare only through its effect on the marginal entrepreneurs:[18] IP increases aggregate welfare by inducing them to change sector of activity. The other entrepreneurs instead never change the sector they invest into.[19] Since the higher welfare gain is generated by the reallocation of entrepreneurs with lower abilities among those in the

[18] These are the entrepreneurs with ability a_i in the interval ($\bar{a}^n$, $\bar{a}^*$).

[19] These are the entrepreneurs with with $a_i < \bar{a}^n$ or $a_i > \bar{a}^*$. Despite IPA intervention, entrepreneurs with $a_i < \bar{a}^n$ remain in the unknown sector, while those with $a_i > \bar{a}^*$ always stay in the known one.

interval $[b(\pi_u/\pi_k), b(\Pi_u/\Pi_k)]$[20], the effectiveness of the IP on aggregate welfare depends on the population size in that interval. The findings in Figure 3 can then be rationalized noticing that a higher level of b enlarges this interval via the *productivity effect*. At the same time, when $\bar{a}^* > 1/2$ (resp., $\bar{a}^* < 1/2$) a higher inequality tends to reduce (resp., increase) the mass of entrepreneurs in the interval through the *size effect*.

Overall, these results reinforce our previous conclusions: the impact of IP on the economy crucially depends on the distribution of entrepreneurs abilities. This means that the characteristics of the optimal IP vary with the *current* economic conditions. Hence, in order to correctly predict its effects and to minimize unintended negative consequences, a proper understanding of the characteristics of the economic environment - and in particular of the private sector - is a necessary condition for the design of any IP.

4 Robustness and extensions

Until now, we have assumed that the Government is benevolent, it has perfect information, and it has no control over the amount of knowledge concerning the production process in the unknown sector - we have assumed that b is exogenously given. In the followings, we relax these assumptions, showing: 1) the robustness of the basic setup to possible IPA's errors in evaluating the *true* externality of the economic sectors; 2) that, when the Government can influence the level of basic knowledge in the unknown sector, further insights emerge on the effect of IP for the aggregate welfare.

4.1 Optimal IP and Government bias

We first introduce the possibility that a bias affects the Government choice of the optimal IP. This bias can be interpreted - among other possibilities - as a measure of the level of corruption or of imperfect information on the Government side. In particular, we study the effect of a bias in the IPA evaluation of the magnitude of the externality generated in the unknown sector. We assume that while the true value of the unknown sector externality is $\widehat{U}$, the IPA chooses the IP under the expectation that the externality is $U = \widehat{U}\,(1 - z)$, with $z \in [-1,1]$; hence, the lower (resp., higher) the value of z with respect to 0, the more IPA overestimates (resp., underestimates) the true value of the externality. Since the externality produced by the unknown sector can now be smaller than that generated by the known one, we need to abandon the simplifying assumptions we have employed so far, $U > K = 0$

[20] Remember that $\bar{a}^n = b(\pi_u/\pi_k)$ is the ability of the marginal entrepreneur under the neutral policy $\tau_u = \tau_k$, while $\bar{a}^* = b(\Pi_u/\Pi_k)$ is the threshold ability of the marginal entrepreneur under the optimal IP (eq. 7).

(see Section 3.1). This implies that the relationship between the optimal τ_u and τ_k depends on both sector externalities and individual productivities (see eq. 8). In particular τ_u R τ_k, as long as:[21]

$$U \gtreqless \frac{\pi_u}{\pi_k} K \tag{21}$$

In the presence of Government bias, condition (21) becomes:

$$U = \widehat{U}\,(1 - z) \gtreqless \frac{\pi_u}{\pi_k} K \tag{22}$$

To analyze the effects of the IPA bias on aggregate welfare, we compare the aggregate welfare under the optimal IP for different levels of z, and the aggregate welfare in the benchmark case (neutral policy). We define this relative difference as:

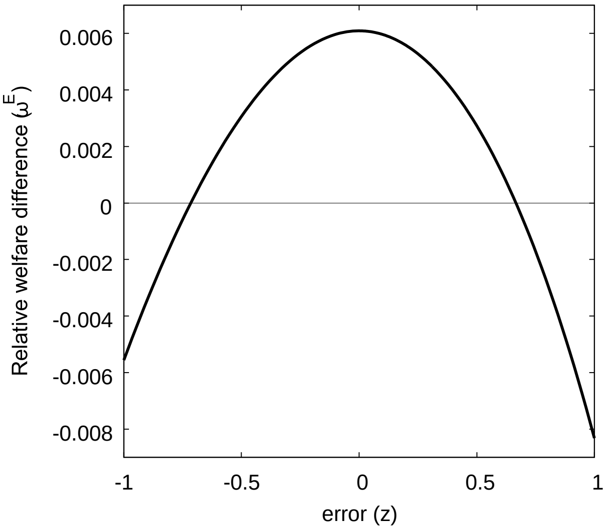
$$\omega^E = \frac{\omega^B}{\omega^n} - 1 \tag{23}$$

with $\omega^B = \omega(\bar{a}^B)$, where $\bar{a}^B$ is the socially optimal allocation of entrepreneurs as defined in eq. (7) under the bias z.

Figure 4 plots ω^E as function of the bias z. Not surprisingly, the larger the bias (either positive or negative) of the IPA the lower the welfare gain of the optimal IP with respect to the neutral policy. More interestingly, the numerical results indicate that there is a non-empty set of values of the bias for which IP is still optimal: there exist configurations of parameters for which IP is optimal even in the presence of a corrupt and non-benevolent Government. Finally, note that the effect of the bias is not symmetric. In particular, underestimating the true value of the externality reduces the welfare gain of IP less than overestimating it. This suggests a conservative approach in estimating the magnitude of the positive externalities generated by investments in the unknown sector as to minimize the possible negative welfare effects of IP in the presence of Government bias.

[21] Condition (21) indicates that it is optimal for the IPA to support relatively more the entrepreneurs in the unknown sector under the condition that U is greater than the weighed K, where the weight is given by the sector specific relative productivities π_u/π_k. Note that the condition is also satisfied when $U < K$ if π_k is sufficiently higher than π_u, $\pi_k >> \pi_u$. This implies that the condition $U > K$ is not necessary to have the Government optimally providing support to the unknown sector.

Figure 4: Relative welfare difference under optimal IP and neutral policy (ω^E) for different level of government bias (z)



Parameters: $\pi_k = 1.5$, $\pi_u = 1$, $U = 1$, $K = 0.5$, $b = 0.2$, $\gamma = 0.7$, $g = 2$.

4.2 *Endogenous sector knowledge level, IPA quality and the effectiveness of Industrial Policy*

Until now, we have assumed that the return for investment projects in the unknown sector y_u is equal for all entrepreneurs, positive and *exogenously* given since it only depends on the sector-level productivity π_u and on the current level of knowledge concerning the production techniques in the unknown sector available in the economy b. However, the level of b - rather than being exogenous - may more realistically depend on a series of industrial policy measures implemented by the IPA. For instance, it may be determined by the quantity of training provided by the IPA to local entrepreneurs planning to invest in the unknown sector; by the quality of the information on markets opportunities provided by the Government to entrepreneurs, etc. It follows that the costs of these activities have to be more realistically included in the IPA budget constraint together with the monetary transfers. Thus, differently from the exogenous case, b is now a choice variable in the Government optimization problem: this implies that the return for the projects in the unknown sector depends on the Government investments to improve the IPA quality. In this sense, b can be interpreted as a proxy for the quality and the capability level of the IPA itself: the higher the ability of the IPA, the higher b, and thus the higher the return from investing in the unknown sector.

In the following, we assume that the cost of achieving a targeted level of b is represented by a specific functional form given by:[22]

[22] In the main text, we highlight only the main features of the modeling strategy. All the mathematical details are included in the Appendix B.

$$c(b) = \frac{1}{2}\Lambda b^2 \tag{24}$$

for some positive constant Λ, representing a technological shift that governs the relevance of the cost function. The IPA budget constraint becomes

$$g = c(b) + \eta\tau_k + (1 - \eta)\tau_u, \tag{25}$$

and the IP becomes defined by the triple (τ_k, τ_u, b), among those satisfying the budget constraint. Given $c(b)$, the sectoral allocation of the entrepreneurs follows the same structure described in Section 2.3: entrepreneurs choose to invest in the unknown sector if and only if the profit is higher than the profit of investing in the known sector (see eq. (3)).[23] Finally, the per-capita welfare function can be rewritten as:

$$\begin{aligned}\omega &= \Pi_k \int_{\bar{a}}^{1} a_i f(a_i)\, \mathrm{d}a_i + \int_{\bar{a}}^{1} \tau_k f(a_i)\, \mathrm{d}a_i + (\Pi_u b + \tau_u) \int_0^{\bar{a}} f(a_i) da_i - c(b) \\ &= \Pi_k \left(\int_{\bar{a}}^{1} a_i f(a_i)\, \mathrm{d}a_i \right) + b\Pi_u \int_0^{\bar{a}} f(a_i) da_i + g - c(b) \end{aligned} \tag{26}$$

As in the case of exogenous b, in the following we compare the social optimal allocation chosen by the IPA with a benchmark. In this context, to be consistent with our previous analysis, we consider as a benchmark a situation where the level of b is exogenously fixed and $\tau_u = \tau_k = \frac{G-c(b)}{M}$. It is immediate to show that Proposition 3.2 holds also in this new setting (see Proposition B.1).

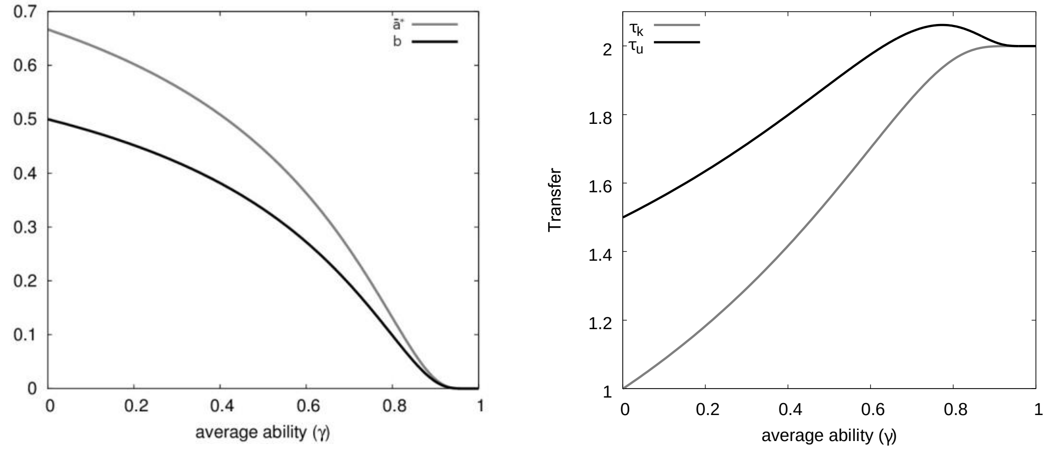
4.2.1 IP and entrepreneurs' quality

IP, welfare and average ability As in the exogenous case, the characteristics of the optimal IP depend on the average entrepreneurs' abilities (Figure 5). The numerical results in Panel 5a shows that, if we choose the same cumulative distributions F_γ as in Section 3.2, b (the optimal level of IPA quality chosen by the Government) and $\bar{a}^*$ (the ability of the marginal entrepreneur under the optimal IP) are both decreasing functions of γ (the average level of abilities in the entrepreneurs population). These results complement those derived in Section 3.2: a higher average ability in the population is optimally associated with larger transfers to support projects in the known sector (Panel 5b) and with smaller investment to support IPA activities. This can be interpreted as suggesting that the ability level of entrepreneurs and that of the IPA are in fact substitutes: the higher is the ability of the private sector, the less useful are investments to improve the capabilities of the IPA in coordinating or guiding the economic activity. Moreover, Panel 5b shows that, differently from the exogenous case, now the

[23] The only difference is that now b in equation (3) is a choice variable for the Government.

difference between τ_k and τ_u is not longer constant across entrepreneur's average ability levels,[24] and the value of τ_u is not necessarily monotonic.[25]

Figure 5: Optimal IPA quality (b), share of entrepreneurs in the unknown sector ($\bar{a}^*$) and individual sector-specific transfers as function of the entrepreneurs' average ability (γ)



(a) b and $\bar{a}^*$ (b) Individual transfers

Parameters: $\pi_k = \Pi_k = 1.5$, $\pi_u = 1$, $U = 1$, $g = 2$, $\Lambda = 4$

Finally, similarly to the exogenous case, the effect of the optimal IP is always welfare increasing and its effectiveness depends on γ. The mechanics is the same as for Figure 1: the effect of IP is small for very low and very high levels of average ability because in these cases the entrepreneur's individual choice in the benchmark is similar to that under the optimal IP. In the case of endogenous IPA quality, this effect is even stronger because for higher levels of γ the optimal level of b is lower.

[24] In the exogenous setting (eq. 9), the difference in the individual transfers for the entrepreneurs in the two sectors $\tau_u - \tau_k$ is constant and determined only by exogenously fixed parameters (i.e., b,U,K,π_k,π_u). It follows that, whatever the ability level, also the difference between the optimal individual incentives is fixed.

[25] We have that

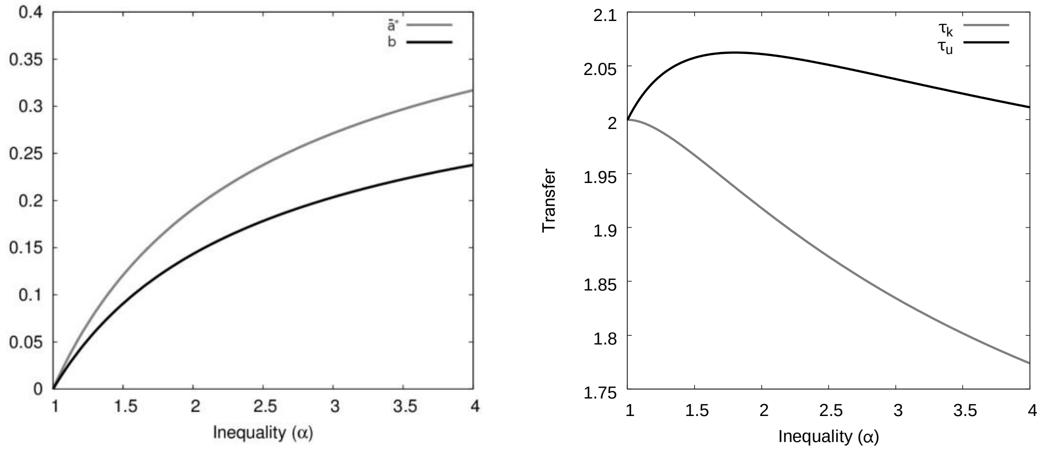
$$\tau_u = \Omega - \frac{1}{2}\Lambda b^2 + \left(1 - F\left(\frac{\Pi_u b}{\Pi_k}\right)\right) bU.$$

with b being the optimal level of IPA quality, as represented in Figure 5a. The term $\frac{1}{2}\Lambda b^2$ is the cost of maintaining the IPA structure: the higher the cost to maintain the IPA the less the resources available to be transferred to entrepreneurs investing in the projects in unknown sector (and in the known sector as well, since the same term appears also in the expression of τ_k). The term $\left(1 - F\left(\frac{\Pi_u b}{\Pi_k}\right)\right) bU$ instead represents the value of the "missed externalities". It is given by the product between $\left(1 - F\left(\frac{\Pi_u b}{\Pi_k}\right)\right)$ (which is the number of projects activated in the known sector, i.e. the sector without positive externality), U (which is the intensity of the externality) and b. In other words, this is the value the projects activated in the known sector would have had if they had been activated in a unknown sector. While the behavior of the cost $\frac{1}{2}\Lambda b^2$ is monotonic in γ (we have already observed that b always decreases when γ increases), the behavior of the missed externalities is not monotonic. Indeed the optimal number of projects in the known sector $\left(1 - F\left(\frac{\Pi_u b}{\Pi_k}\right)\right)$ increases with the ability level of the population while the optimal quality b of the IPA structure decreases. Panel 5b shows that the first effect is stronger for small values of γ while the second is stronger for γ large enough. This non-monotonic behavior of the missed externalities causes the non-monotonic behavior of τ_u.

IP, welfare and inequality levels in the ability distribution To analyze how abilities' distributions characterized by different inequality levels affect the optimal level of IPA quality, the optimal share of entrepreneurs in the unknown sector, and aggregate welfare, we consider the same family of cumulative distribution described in section 3.2 (eqs. 18-19).

Numerical solutions of the model are illustrated in Figures 6 and 7.[26] Panel 6a indicates that the higher is the inequality in the abilities' distribution (higher α), the higher are both the optimal level of IPA quality (b) and the ability of the marginal entrepreneur ($\bar{a}^*$). Indeed, the higher the inequality, the larger the share of the population with very low ability, and thus the higher the potential gain that can be achieved by training and employing them in the unknown sector. Again, this suggests that the IPA intervention is particularly important in economies characterized by higher abilities' inequality: even if costly, the activity of the IPA is optimal because it allows the identification and support of projects in the unknown sector in a situation in which the entrepreneurs' abilities are particularity weak.

Figure 6: Optimal IPA quality (b), productivity of the marginal entrepreneur ($\bar{a}^*$) and optimal transfers as function of of the distribution of entrepreneurs' ability (α)



(a) b and $\bar{a}^*$ (b) Individual transfers

Parameters: $\pi_k = \Pi_k = 1.5$, $\pi_u = 1$, $U = 1$, $g = 2$, $\Lambda = 4$

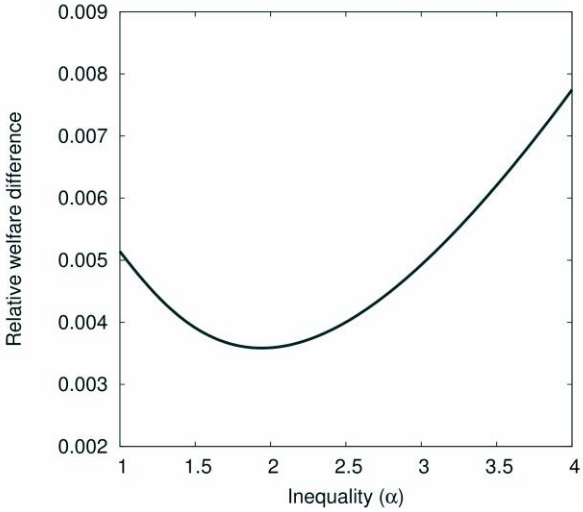
As shown in Panel 6b, this mechanism affects also the individual monetary resources that the IPA can optimally transfer to the entrepreneurs. Increases in the spread of the abilities' distribution induces the IPA to invest more to improve the quality of services provided (as measured by b), which, as a consequence, partially crowds out the individual transfers. Hence, *both* the individual transfers tends to decrease when the level of the inequality increases, indicating that the higher the inequality, the stronger the incentives of the IPA to substitute improvements in quality of services for higher

[26] Mathematical details are sketched in the Appendix B.

individual transfers. However one can observe that, consistently with the described behavior of b, the net incentive $\tau_u - \tau_k$ to switch to the unknown sector increases when α increases and that the individual transfers to entrepreneurs in the unknown sector are not necessarily monotonic, increasing for very low levels of inequality and decreasing only for high levels of inequality. This latter effect depends on the quality of the support that the IPA can offer to entrepreneurs investing in the unknown sector. When the inequality in the ability of the entrepreneurs is very low, it is optimal for the IPA to invest less in the quality of services provided (i.e. choosing a low b). As a consequences, for low levels of inequality, the IPA substitutes monetary transfers for the provision of services to sustain the activity of the unknown sector entrepreneurs: this implies that individual transfers to entrepreneurs in the unknown sector are larger. On the contrary, when the quality of the IPA (b) is large enough, the resource constraint makes it optimal for IPA to reduce the individual transfer to entrepreneurs in the unknown sector.

Finally, results reported in Figure 7 show that the welfare gain associated with the optimal IP is non-monotonic in the levels of inequality in the ability distribution. This is due to two different effects of the IP. On one hand, there is the same mechanism described in Figure 1: the higher the polarization the more similar the individual and the optimal social choice. This implies that the effectiveness of the IP is smaller for higher values of inequality. On the other hand, the higher the inequality the higher the optimal level of training and services provided by IPA (as measured by b) and then the larger the benefit of re-allocating workers. Hence, IP turns to be most effective when inequality is either very small or very high.

Figure 7: Welfare gain under the optimal IP for different levels of inequality in the ability distribution (α).



Parameters: $\pi_k = \Pi_k = 1.5$, $\pi_u = 1$, $U = 1$, $g = 2$, $\Lambda = 4$.

5 Conclusions

In this paper we have provided a simple model to analyse the effects of IP on aggregate welfare. The setting of the model is able to capture some of the peculiar features of most developing economies and allows us to derive a number of results. In particular, we have shown how the optimal IP changes with the ability distribution of the entrepreneurs' population and the Government bias.

Our results support the view that that there is not a one-for-all optimal IP but rather there is an IP that is more likely to be most effective given a certain distribution of abilities among entrepreneurs in any specific historical moment. In fact, our results show that the same intervention (a simple cash transfer) may have very different (even opposite) effects depending on the distribution of entrepreneurs capabilities.

These results also shed some lights on the reasons for the very different effects that the various development strategies implemented in the last four decades (i.e. inward-industrialization strategy, structural adjustment programs, etc.) had in different countries. While we do not discuss why these waves of development strategies are adopted in developing countries (on this see Hodler, 2011), we argue that these strategies cannot be said to be wrong or correct in abstract since the effects of Government intervention (or lack thereof) ultimately depends on the distribution of entrepreneurs abilities and Government quality. Our results thus suggest that there is no much sense in trying to identify the best development strategy and make developing countries to adopt that, as it is still too much common in development advocacy. In fact, Government interventions that have been historically effective (for instance in the form of Industrial Policy) had a strong country-specific component and had been shaped according to the characteristics of the entrepreneurial class and government capabilities and on how they historically evolved creating the current economy context. We leave the exploration of these propositions for further research.

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Appendices

A Proofs and technicalities for Section 3

The following more general version of Proposition 3.1, that also takes into account the corner solutions case, can be obtained using a standard maximization procedure.

Proposition A.1. *The optimal IP is characterized by non-negative individual transfers* τ_u *and* τ_k *that satisfy the budget constraint in eq.* (1). *The optimal individual transfers can be characterized as*

(i) If $\left(\frac{U\pi_k - K\pi_u}{\pi_k + K}\right) \in \left(-\frac{g}{\left(1-F\left(\frac{\Pi_u b}{\Pi_k}\right)\right)b}, \frac{g}{F\left(\frac{\Pi_u b}{\Pi_k}\right)b}\right)$ *then the optimal* τ_k *and* τ_u *are*

$$\tau_u = g + (1 - F(\bar{a}^*))\, b \left(\frac{U\pi_k - K\pi_u}{\pi_k + K}\right) \tag{A.1}$$

$$\tau_k = g - F(\bar{a}^*)\, b \left(\frac{U\pi_k - K\pi_u}{\pi_k + K}\right) \tag{A.2}$$

(ii) If $\left(\frac{U\pi_k - K\pi_u}{\pi_k + K}\right) \leq -\frac{g}{\left(1-F\left(\frac{\Pi_u b}{\Pi_k}\right)\right)b}$*, then the optimal* τ_k *and* τ_u *are*

$$\tau_u = 0 \tag{A.3}$$

$$\tau_k = g + F\left(\frac{\Pi_u b}{\Pi_k}\right)\frac{g}{\left(1 - F\left(\frac{\Pi_u b}{\Pi_k}\right)\right)} = \frac{g}{\left(1 - F\left(\frac{\Pi_u b}{\Pi_k}\right)\right)} \tag{A.4}$$

(iii) If $\left(\frac{U\pi_k - K\pi_u}{\pi_k + K}\right) \geq \frac{g}{F\left(\frac{\Pi_u b}{\Pi_k}\right)b}$*, then the optimal* τ_k *and* τ_u *are*

$$\tau_u = g + \left(1 - F\left(\frac{\Pi_u b}{\Pi_k}\right)\right)\frac{g}{F\left(\frac{\Pi_u b}{\Pi_k}\right)} = \frac{g}{F\left(\frac{\Pi_u b}{\Pi_k}\right)} \tag{A.5}$$

$$\tau_k = 0. \tag{A.6}$$

Proof of Lemma 3.3. We have that $F_1(\bar{a}^*) = F_1\left(\frac{\Pi_u b}{\Pi_k}\right) \geq F_2\left(\frac{\Pi_u b}{\Pi_k}\right) = F_2(\bar{a}^*)$ so $\eta_1 = 1 - F_1(\bar{a}^*) \leq 1 - F_2(\bar{a}^*) = \eta_2$. □

Proof of Proposition 3.4. From equations (12), (13) and (15) it derives that a decrease in the proportion of entrepreneurs in the unknown sector $F(\bar{a}^*)$, induced by an increase in the average ability γ, implies an increase of τ_u, τ_k and T_k. Hence, from the budget constraint (1), T_u decreases correspondingly. Part (ii) describes just the corner solution case. □

B Proofs and details for Section 4.2

The objective of this section is to formally characterize the optimal IP when the IPA may use - in addition to individual transfer - also an additional set of costly non-monetary instruments, represented by the level of b.

Define $c(b)$ the function (increasing in b and with $c(0) = 0$) that describes the per-capita cost for the IPA to generate a level of basic knowledge in the unknown sector b for the projects in the unknown sector. Later $c(b)$ will be specified as in Section 4.2. As already observed in Section 4.2, in this case the (per-capita) IP budget constraint becomes:

$$g = c(b) + \eta\tau_k + (1 - \eta)\tau_u. \tag{B.1}$$

and the per-capita welfare function is:

$$\begin{aligned}\omega = \Pi_k \int_{\bar{a}}^{1} a_i f(a_i)\,\mathrm{d}a_i + \int_{\bar{a}}^{1} \tau_k f(a_i)\,\mathrm{d}a_i + (\Pi_u b + \tau_u)\int_0^{\bar{a}} f(a_i)da_i - c(b) \\ = \Pi_k \left(\int_{\bar{a}}^{1} a_i f(a_i)\,\mathrm{d}a_i\right) + b\Pi_u \int_0^{\bar{a}} f(a_i)da_i + g - c(b). \quad \text{(B.2)}\end{aligned}$$

The IPA choose (τ_k,τ_u,b) to maximise eq. (B.2)[27]. For any possible choice of b, the maximization problem in the variables τ_k and τ_u is the same of Section 3 so the optimal individual transfers are given by:

$$\tau_u = \quad g - c(b) + \left(1 - F\left(\frac{\Pi_u b}{\Pi_k}\right)\right) bU \tag{B.3}$$

$$\tau_k = \quad g - c(b) - F\left(\frac{\Pi_u b}{\Pi_k}\right) bU \tag{B.4}$$

and then, again as in Section 3, the level of ability that discriminates between entrepreneurs investing in the known or in the unknown sector is:

$$\bar{a}^* = \frac{\Pi_u b}{\Pi_k} \tag{B.5}$$

It follows that welfare maximization becomes a one-dimensional problem. Hence, using (B.5), (B.2) it can be rewritten as:

$$\Pi_k \int_{\frac{\Pi_u}{\Pi_k}b}^{1} af(a)\,\mathrm{d}a + \Pi_u \int_0^{\frac{\Pi_u}{\Pi_k}b} bf(a)\,\mathrm{d}a + g - c(b) \tag{B.6}$$

By maximizing (B.6), we find that $b \in (0,1)$ is a critical point if and only if

[27] In the following, we assume that G is large enough to allow for interior maxima of τ_k and τ_u (both strictly positive).

$$\Pi_u F\left(\frac{\Pi_u}{\Pi_k}b\right) = c'(b) \tag{B.7}$$

Observe that since $c'(0) > 0$ (i.e. the cost of improving IPA quality is increasing in the quality), the optimal IP always requires $b > 0$.[28]

As in the case of exogenous *b*, our next step is to compare the social optimal allocation chosen by the IPA with a "neutral" benchmark. In this context, to be consistent with our previous analysis, we consider as a benchmark the neutral policy case, i.e. a situation where the level of *b* is exogenously fixed and $\tau_u = \tau_k = \frac{G-c(b)}{M}$. From equation B.7, it immediately follows that:

Proposition B.1. *Aggregate welfare under optimal IP is always larger than under the neutral policy.*

B.1 IP and entrepreneurs' quality (Section 4.2.1)

Comparing populations of entrepreneurs with different average ability levels In this paragraph we show how to derive the results of the paragraph *IP, welfare and average ability* in Section 4.2.1. As in that section we assume that the cost function for the IPA is quadratic of the form:

$$c(b) = \frac{1}{2}\Lambda b^2 \tag{B.8}$$

We consider the same family of cumulative distributions we used in the Section 3.2:

$$F_\gamma(a) = a^\gamma$$

for $\gamma \in (0,1)$. Finally, we make a technical assumption to ensure an internal maximum, namely:

$$\Lambda > \frac{\Pi_u^2}{\Pi_k}. \tag{B.9}$$

Under these assumptions, the model can be analytically solved. The results are illustrated in the following proposition. They are used in the numerical study of Section 4.2.

Proposition B.2. *The optimal IP is characterized by the following triple:*

[28] If $b = 0$ was optimal, it would satisfy (B.7) and on the left side of the equation we would have $\Pi_u F(0) = 0$.

$$
\begin{cases}
b_\gamma = \left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{1}{1-\gamma}} \\
\tau_{u,\gamma} = g - \frac{1}{2}\Lambda\left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{2}{1-\gamma}} + \left(1 - F\left(\frac{\Pi_u\left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{1}{1-\gamma}}}{\Pi_k}\right)\right)\left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{1}{1-\gamma}} U \\
\tau_{k,\gamma} = g - \frac{1}{2}\Lambda\left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{2}{1-\gamma}} - F\left(\frac{\Pi_u\left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{1}{1-\gamma}}}{\Pi_k}\right)\left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{1}{1-\gamma}} U.
\end{cases}
$$

The corresponding per-capita welfare level is given by:

$$
\omega = \Pi_k \frac{\gamma}{\gamma+1}\left(1 - \left(\frac{\Pi_u^2}{\Pi_k \Lambda}\right)^{\frac{\gamma+1}{1-\gamma}}\right) + \Pi_u \left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{1}{1-\gamma}} \left(\frac{\Pi_u^2}{\Pi_k \Lambda}\right)^{\frac{\gamma}{1-\gamma}} + g.
$$

Proof. For the specific case considered here, (B.7) becomes

$$
\Pi_u \left(\frac{\Pi_u}{\Pi_k} b\right)^\gamma = \Lambda b.
$$

There exists a unique strictly positive solution b_γ of such an expression and, thanks to (B.9), it belongs to (0,1). It is given by

$$
b_\gamma = \left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{1}{1-\gamma}}
$$

Using standard arguments, it can be easily verified that such a critical point, whenever interior (as in the simulation we present), is in fact a maximum. The corresponding value of $\bar{a}^*$, found using (B.5), denoted with $\bar{a}_\gamma$, is given by

$$
\bar{a}_\gamma = \left(\frac{\Pi_u^2}{\Pi_k \Lambda}\right)^{\frac{1}{1-\gamma}}
$$

To find the corresponding values of τ_k and τ_u one has only to use (B.3). The explicit expression of the net welfare is obtained using the previous values in the expression of the welfare:

$$
\begin{aligned}
\omega = \Pi_k \int_{a \geq \bar{a}^*} a f(a)\,\mathrm{d}a + \Pi_u \int_{a < \bar{a}^*} b f(a)\,\mathrm{d}a + g - \frac{1}{2}\Lambda b^2 \\
= \Pi_k \int_{\left(\frac{\Pi_u^2}{\Pi_k \Lambda}\right)^{\frac{1}{1-\gamma}}}^{1} a \gamma a^{\gamma-1}\,\mathrm{d}a \\
+ \Pi_u \int_0^{\left(\frac{\Pi_u^2}{\Pi_k \Lambda}\right)^{\frac{1}{1-\gamma}}} \left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{1}{1-\gamma}} \gamma a^{\gamma-1}\,\mathrm{d}a + g - \frac{1}{2}\Lambda \left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{2}{1-\gamma}} \\
= \Pi_k \frac{\gamma}{\gamma+1}\left(1 - \left(\frac{\Pi_u^2}{\Pi_k \Lambda}\right)^{\frac{\gamma+1}{1-\gamma}}\right) + \Pi_u \left(\frac{\Pi_u^{\gamma+1}}{\Pi_k^\gamma \Lambda}\right)^{\frac{1}{1-\gamma}} \left(\frac{\Pi_u^2}{\Pi_k \Lambda}\right)^{\frac{\gamma}{1-\gamma}} + g. \quad \text{(B.10)}
\end{aligned}
$$

This concludes the proof. □

IP, welfare and inequality In this paragraph we show how to derive the results of the paragraph *IP, welfare and inequality levels in the ability distribution* in Section 4.2.1. As mentioned in Section 4.2.1 we use in this case the same cumulative distribution function introduced in (18) and we specify the cost function $c(b)$ as in (B.8). The optimum triple $(b_\alpha, \tau_{k,\alpha}, \tau_{u,\alpha})$ can be then explicitly characterized as shown in the following proposition. This result is used in the numerical study of Section 4.2.

Proposition B.3. *The optimal IP is characterised by the following triples* $(b_\alpha, \tau_{k,\alpha}, \tau_{u,\alpha})$*:*

$$\begin{cases} b_\alpha = \frac{\Pi_k}{\Pi_u}\bar{a}_\alpha \\ \tau_{u,\alpha} = g - \frac{1}{2}\Lambda\left(\frac{\Pi_k}{\Pi_u}\bar{a}_\alpha\right)^2 + (1 - F(\bar{a}_\alpha))\frac{\Pi_k}{\Pi_u}\bar{a}_\alpha U \\ \tau_{k,\alpha} = g - \frac{1}{2}\Lambda\left(\frac{\Pi_k}{\Pi_u}\bar{a}_\alpha\right)^2 - F(\bar{a}_\alpha)\frac{\Pi_k}{\Pi_u}\bar{a}_\alpha U. \end{cases}$$

where

$$\bar{a}_\alpha = \frac{\frac{3}{2}\alpha - \sqrt{\frac{9}{4}\alpha^2 - 4\alpha\left(\frac{\alpha}{2} + 1 - \Lambda\frac{\Pi_k}{\Pi_u^2}\right)}}{2\alpha}.$$

Proof. In this case, (B.7) specifies as

$$\Pi_u F\left(\frac{\Pi_u}{\Pi_k}b\right) = \frac{\Pi_u^2}{\Pi_k}b\left(\alpha\left(\frac{\Pi_u}{\Pi_k}b - 1/2\right)\left(\frac{\Pi_u}{\Pi_k}b - 1\right) + 1\right) = \Lambda b$$

that can be solved for $\bar{a}^* = \frac{\Pi_u}{\Pi_k}b$ obtaining

$$\bar{a}^* = \frac{\frac{3}{2}\alpha \pm \sqrt{\frac{9}{4}\alpha^2 - 4\alpha\left(\frac{\alpha}{2} + 1 - \Lambda\frac{\Pi_k}{\Pi_u^2}\right)}}{2\alpha}$$

By standard arguments one can easily see that the solution

$$\bar{a}^* = \frac{\frac{3}{2}\alpha - \sqrt{\frac{9}{4}\alpha^2 - 4\alpha\left(\frac{\alpha}{2} + 1 - \Lambda\frac{\Pi_k}{\Pi_u^2}\right)}}{2\alpha}$$

if interior, is a maximum point of the welfare. Give such a $\bar{a}^*$ we can find the explicit formulas for τ_u and τ_k using (B.3) and (B.4). □